Endogenous Industrial Cycles in a Reshaped "Neoclassical" Model

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Abstract

Two closely related mathematically sophisticated "neoclassical" models of economic growth (first with hidden, second, more general, with intended) economies of scale are considered. The main variables are relative wage and employment ratio, whereas a ratio of investment to profit is constant. The efficiency wage hypothesis supports equations for a growth rate of output per worker. Workers' competition for jobs is stabilizing and their fight for increased wages is destabilizing as revealed. In each model, a stationary state is locally asymptotically stable in a system of two ODEs. There is no possibility for endogenous industrial cycle. Reality requires negation of this incorrect denial.

A third extended model, containing the greed feedback loops, reflects destabilizing cooperation and stabilizing competition of investors. In a system of three ODEs, rate of capital accumulation becomes the new main variable. Its targeted long-term decrease raises profit rate together with reducing relative wage and capital-output ratio. Oscillations imitating industrial cycles are endogenous. Crisis is a manifestation of relative and absolute over-accumulation of capital. Limit cycle with a period of about 6.75 years results from supercritical Andronov – Hopf bifurcation. The reality disagrees with the efficiency wage hypothesis applied in the analyzed models.

Keywords: rate of capital accumulation, relative wage, employment ratio, efficiency wage hypothesis, industrial cycle, supercritical Andronov – Hopf bifurcation, limit cycle

To Sesquicentennial (2017) of K. Marx'
Capital: Critique of Political Economy, vol. I.

1. Introduction on K. Marx' industrial cycles and their denial in modern "neoclassical" models

The present paper considers the notion of industrial cycle as a dynamic process typical for *industrial capital* with a specific turnover in a macroeconomic context without disaggregating national economy in industry and other sectors. According to this understanding based on the first, second and third volumes of K. Marx "Capital", *industrial capital* moves in such diverse (still interconnected) fields as industry, agriculture, construction and other sectors.[1] The notion industrial capital is mostly restricted in this paper to the notion of *productive capital* as the single source of surplus product and surplus value in capitalist economy. Uncovering specific regularities of industrial cycle in industry and other sectors of capitalist economy will require additional efforts far beyond restricted scope of this paper.

The term industrial cycle rooted in the K. Marx works became out of fashion. It was substituted by such terms as trade cycle and business cycle. The present paper reintroduces the original term with intent to stimulate the reader interest in the Marxist economic theory. "The course characteristic of modern industry, viz., a decennial cycle (interrupted by smaller oscillations), of periods of average activity, production at high pressure, crisis and stagnation, depends on the constant formation, the greater or less absorp-

[1] An attempt to restrict the notion of industrial cycle to fluctuations in indicators of industry would be a wrong denial of *universal* (or *general*) over-production and mistakenly viewing this over-production as basically partial (in particular sectors, spheres or different branches of production).

tion, and the re-formation of the industrial reserve army or surplus population… Effects, in their turn, become causes, and the varying accidents of the whole process, which always reproduces its own conditions, take on the form of periodicity." [1: 419–420].

K. Marx assumed in 1870, on the basis of the recurrence of crises in 1847, 1857 and 1867 that the period of the industrial cycle was ten years. Later he stressed that the period of the cycle was likely to shorten as the pace of technical progress increased (see for details [2]).

The K. Marx economic theory infers from the laws of capitalist production that crises are the most dramatic manifestation of the underlying contradictions of the capitalist system, they manifest themselves through apparently contingent causes of each individual crisis [2].

It has been the shocking revelation for ordinary citizen by an analysis company that "economists, as a consensus, called exactly none" of the last seven recessions, dating back to 1970. In other words [3], the consensus view of economists has an accuracy rate of zero per cent: "…economists seem to be motivated by a fear of bucking the professional consensus. Their models are not built to capture dramatic shifts and their forecasts tend to move in tiny steps."

One case is typical: in fall 2008 when the recession was already unfolding in the USA for more than half a year even the journal of the Union for Radical Political Economics published a paper [4] endorsing an implicit claim that trade cycles are the thing of the past in an influential paper of a renowned "neoclassical" economist [5].

Anticipating (or reflecting) next global recession it would be wise to investigate the logic of both papers hidden behind conformism ("a fear of bucking the professional consensus"). The system dynamics methodology is a promising tool for such endeavour. J. Forrester [6: 362, 370] wrote: "There is lack of courage in the field to open oneself to severe debate and criticism. However, we will never change intended and widely supported detrimental policies without intense debate ... System dynamicists must go behind the symptoms of trouble and identify the basic causes."

The "neoclassical" school falls short of these essential requirements. Dialectic (historic) materialism is deeper than subjectivism (alleged objectivism) of the "neoclassical" school.

F. van der Ploeg restructured the famous Goodwin version [7] of the Marxist growth cycle. He concluded [5: 228]: "When the assumption of a fixed capital-output ratio is replaced by the assumption that firms maximize profits subject to a C.E.S. production function, whilst retaining the wage-bargaining equation of R. Goodwin (1967), the perpetual class struggle cycles of Goodwin's model are replaced by either damped conflict cycles or monotonic convergence to balanced growth equilibrium".[2]

Still F. van der Ploeg [5: 229–230] recommends for a more realistic theory allowing natural and warranted growth rate to differ by introducing an independent investment function taking into account profitability and other factors.

The efforts in transforming the perpetual cycles of R. Goodwin (1967) into damped conflict cycles by "neoclassical" tools have been supported by L. Aguiar-Conraria [4]. He undoubtedly tried a step in the right direction to "endogenizing productivity growth". This paper reveals the strength and the weakness of [4] that contains grains of truth that need refinement and extension.

Unfortunately, L. Aguiar-Conraria [4] is very misleading taking it as a whole. Instead of following F. van der Ploeg's recommendation of introducing a realistic investment function, the assumption on constancy of the accumulation rate has not been relaxed, the destabilizing effects of endogenous productivity growth have been treated superficially, thoughtful remarks [5: 229] on decisive role of workers' competition for jobs for asymptotically stable growth trajectories have not been elaborated. On other hand, important shortcomings of F. van der Ploeg's [5] have been amplified. Such papers do not advance Marxian economics, listed among the keywords in [4], contrary to our expectations.

The reader well remembers that the original Phillips nominal wage-change equation in [9] is nonlinear where factors are the employment ratio, its rate of change and the rate of inflation. R.M. Goodwin [7] simplified this equation in his original model to a linear one for a rate of change of real wage depend-

[2] Starting with a rather problematic quasi-empirical relation between net output per worker and real wage only (cf. equation (35) in section 2.2), a CES production function was derived in [8]. Hardly anybody reviews its innumerable statistical applications in spite of fundamental specification errors.

ent only on a single variable – the employment ratio.

F. van der Ploeg [5] applied a non-linear Phillips equation similar to the linear counterpart used by R. Goodwin [7], besides that a new linear term was added which reflects the growth rate of output per worker as a factor in real wage-change equation. This term implies *ceteris paribus* that the higher growth rate of output per worker compels capitalists to promote a growth rate of real wage that is detrimental for profit and profitability. L. Aguiar-Conraria [4] simply removed this hypothetic term without explanation.

This paper's sections 3.2, 4.1 and 4.2 state a number of propositions and corollaries. Appendix A contains formal proofs of three most complicated ones only. Throughout this text a variable's time derivative is denoted by a dot, its growth rate – by a hat over the variable's sign. An ordinary differential equation is shortened as ODE.

The rest of this paper is organized along the following main themes.

Section 2 reviews briefly properties of the Goodwin model abbreviated as M-1. F. van der Ploeg dynamic model based on a "neoclassical" CES production function [5] denoted as P-1 is considered next. The conservative oscillations in M-1 are stabilised in P-1 mostly due to introduced workers' competition for jobs. More technically, an intensive form of P-1 is built into a two-dimensional system of non-linear ODEs, local asymptotical stability of its non-trivial stationary state is exposed. The stationary employment ratio is lower than that in M-1 because of the surmised direct dependence of the growth rate of wage on growth rate of output per worker.

Section 3 is devoted to L. Aguiar-Conraria model [4] denoted P-2 as extension of P-1. P-2 with additional scale effects provides check of P-1 structural stability. Proportional and derivative control over capital accumulation, implicit in P-2, is revealed. As extension of P-1, P-2 reflects workers' joint struggle for higher relative wage that does not destroy the local asymptotical stability of a stationary state. Conceptual weakness of P-2 resulting from the efficiency wage hypothesis and other "neoclassical" beliefs is uncovered.[3]

Section 4 offers a modernized model Z-1 that extends P-2 through Marx' implicit balancing feedback loop involving endogenous rate of capital accumulation. Z-1 sheds light on industrial cycles as, first of all, capital accumulation cycles. It is shown that capitalists' investment cooperation weakens (competition strengthens) stability of a stationary state in Z-1. Targeted reduction of the stationary rate of accumulation increases stationary profit rate and output-capital ratio, yet it reduces stationary labour value. Self-sustained industrial cycles are born in result of a super-critical Andronov – Hopf bifurcation. Dual nature of capital as the driver and barrier of capitalist production is demonstrated analytically and numerically through Vensim simulation runs.

Section 5 concludes.

2. How conservative oscillations in Goodwin's model (M-1) were "neoclassically" stabilised

2.1. The M-1 extensive and intensive forms

This subsection shortly reviews famous Goodwin's predator-prey model for closed economy [7]. Labourers are advancing capitalists as they receive wage after a particular circuit of capital is finished. Having abstracted from the public sector and foreign economic relations, M-1 consists of the following equations:

$$q = k/s_0; \tag{1}$$

$$a = q/l; \tag{2}$$

[3] The efficiency wage hypothesis has its origins in the controversial logarithmic equation for net output per worker linearly and positively depending only on real wage [8: 228]. Adding capital intensity is this equation has been a standard way for deriving a typical variable elasticity of substitution (VES) production function in the "neoclassical" literature [10]. We will see that a VES production function in [4] has been implicitly derived through direct additional dependence of the growth rate of output per worker on the growth rate of fixed production assets instead of the growth rate of capital intensity (equation (49) in section 3.2). Still the "marginal productivity" principle of income distribution is broken in [4] without noticing by its author. See section 3.2.

$$u = w/a, \tag{3}$$

$0 < u < 1$;

$$\hat{a} = \alpha = \text{const} > 0, \tag{4}$$

$$s_0 = \text{const} > 1; \tag{5}$$

$$v = l/n, \tag{6}$$

$0 < v < 1$;

$$n = n_0 e^{\beta t} \tag{7}$$

$\beta = \text{const} \geq 0$, $n_0 > 0$;

$$\hat{w} = f(v), \tag{8}$$

$$q = C + \dot{k} = wl + (1 - z)M + \dot{k}; \tag{9}$$

$$\dot{k} = zM = z(1 - u)q, \tag{10}$$

$(\alpha + \beta)s_0 < z \leq 1$.

Equation (1) specifies a technical-economic relationship between fixed capital k and net product q.

Capital-output ratio is denoted by s_0. Equation (2) expresses output per worker a as a ratio of net product q to employment l. Equation (3) describes relative wage as wage share in net product u. Equation (4) assumes a constant exogenous growth rate of output per worker α that equals to a growth rate of capital intensity k/l, whereas capital-output ratio remains constant according to equation (5).

Equation (6) defines employment ratio v as a result of the sale of the labour power. According to (7), the growth rate of labour force n is equal to constant β. Equation (8) links the growth rate of real unit wage w with employment ratio v.

The use of current profit reflects absence of information lags for labourers regarding the actual relative wage. In other words, capitalists and workers receive information on relative wage in real time.

Balance equation (9) shows the end use of net product q, where C is non-productive consumption, $\dot{k}$ is net fixed capital formation defined in the equation (10). Investment delays as well as discrepancies between orders and inventories are not taken into explicit account. In result, net fixed capital formation equals net investment. The attribute 'net' will be omitted, as a rule, below for brevity.

Surplus product that equals total profit M can be not only invested, but also be used to cover personal expenses of the bourgeoisie and via implicit taxes for unspoken public consumption. Consequently, rate of accumulation z, measured as share of investments in surplus product, or as ratio of investment to profit, is such that $(\alpha + \beta)s_0 < z \leq 1$. The left boundary is set to avoid a non-positive stationary relative wage (see equation (13)). Notice that $z = 1$ originally in M-1, P-1 and P-2. The given definition of the rate of accumulation z differs from defining it as a growth rate of fixed capital in [16: xvi] and elsewhere.

The presence of $(\alpha + \beta)s_0$ as a lower boundary for rate of accumulation z is a drawback of M-1 and subsequent models, since in reality relative wage remains positive even when $(\alpha + \beta)s_0 \geq z$. This means they do not pass this particular extreme condition test. Models in [11–13] contain endogenous capital-output ratio and endogenous rate of accumulation in the absence of the specified lower bound as a real necessity. Long-term decline in this ratio mitigates the tendency of profit rate to fall in the USA, Italy [11–13] and, by the same reason, still hypothetically in other industrialized countries.

According to [7], an intensive form of deterministic M-1 consists of two non-linear ODEs. Here is this system in a generalized form for $0 < (\alpha + \beta)s_0 < z \leq 1$ in relation to the original form (for $z = 1$) and for non-linear Phillips equation as in P-1 and P-2:

$$\dot{u} = [f(v) - \alpha]u, \quad f'(v) > 0; \tag{11}$$

$$\dot{v} = [z(1 - u)/s_0 - (\alpha + \beta)]v. \tag{12}$$

A positive stationary state of the system (11)–(12) is defined as

$$E_G = (u_G, v_G), \tag{13}$$

where $u_G = 1 - \frac{(\alpha+\beta)s_0}{z}$, $v_G = f^{-1}(\alpha)$, $s_0 > 1$.

A stationary growth rate of output per worker and growth rate of wage equals α. A stationary growth rate of fixed capital and net product is $\hat{k}_G = \hat{q}_G = \alpha + \beta$. A stationary rate of surplus value is $m_G' = (1-u_G)/u_G$. A stationary profit rate is $(1-u_G)/s_0 = (\alpha+\beta)/z$, $\alpha > 0$, $\beta \geq 0$.

Figure 1 and Table 1 exhibit a well-known causal loop structure of M-1.

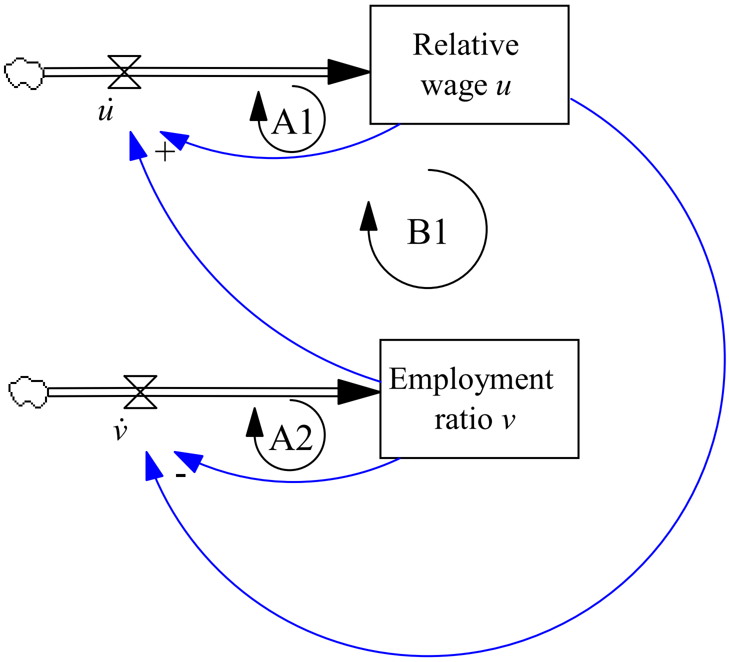

Figure 1 – A condensed causal structure of M-1: total number of feedback loops – 3, among them: 1st order – 2 *alternating*, 2nd order – 1 *negative*

Table 1. The main extensive negative feedback loop in M-1

Loop B1 of length 8
Relative wage $u \xrightarrow{-}$ Profit rate GR fixed capital GR employment ratio Net change of v Employment ratio v GR wage GR relative wage Net change of u

Note. Only a negative first partial derivative is explicitly shown as an arrow. GR is for "Growth rate of".

Improved profitability promotes increases in employment ratio ("prey") that facilitate higher relative wage ("predator") to the detriment of profit rate in negative feedback loop B1.

An equivalent for (15) reflects proportional control over the net change of employment ratio

$$\dot{v} = z\left(\frac{1-u}{s_0} - \frac{\alpha+\beta}{z}\right)v =$$

$$= (z/s_0)(u_G - u)v. \tag{14}$$

Denote derivative of $f(v)$ at v_G as $f'(v)_G$. Typically, the higher is $f'(v)_G$ the faster is adjustment speed of u. Similarly, the higher is z/s_0 the faster is adjustment speed of v. The period of conservative fluctuations in M-1 is closely approximated as

$$T_{M-1} = 2\pi / \sqrt{\frac{(\alpha+\beta) f'(v)_G v_G u_G}{1-u_G}} = 2\pi / \sqrt{u_G \frac{z}{s_0} f'(v)_G v_G} \ . \quad (15)$$

This period clearly depends sensitively on the value of $f'(v)_G$ that can be estimated with less certainty than other relevant parameters. In a simulation run in next section with plausible $f'(v)_G$ and other reasonable magnitudes, T_{M-1} = 21.59 years.

It is relevant to recall A.B. Atkinson's paper that finished with a rather pessimistic conclusion on the relevance of Goodwin's model [14: 151]: "with reasonable values of the parameters, the period is considerably longer than the 4 or 5 years of the typical recent trade cycle. A cycle with this period is only likely to be generated if all profits are saved, if the capital-output ratio is very low (less than 2), and real wages respond very strongly to a fall in unemployment. In fact the model as it stands may be better suited to explaining the 16–22 year "Kuznets" cycle than the post-war trade cycle."

G.W. Low wrote that fluctuations in capital stock, averaging 19 years, reflect the management of investment in fixed capital [15: 337]. Thanks to the G.W. Low critical research this paper will not turn to the inadequate presentation of business cycle in P. Samuelson's multiplier-accelerator model proposed in the late 1930-s and modified subsequently.[4]

G.W. Low's extended system dynamics model suggests that labour and inventory management can produce short-term cycles independently of the long-term fluctuations observed in fixed capital. Still G. W. Low has used noise input to keep business cycles going in that model for interaction of labour force and inventories. Although his model contains objective elements it is not a required solution of the problem of endogenous business cycles modelling.

The present paper steps back in relation to G.W. Low's research abstracting from inventories because there are deeper reasons than disequilibrium on product market for business cycles: the contradiction between social character of production and private capitalist ownership on the means of production and contradiction between value and use-value of labour power. These two fundamental contradictions are to be taken into account before their ramifications closer to the surface of production relations can be modelled as subordinated processes (including fluctuations in inventories and orders).

G.W. Low [15] helps in filtering out unsuccessful attempt to solve the problem of business cycle modelling proposed in [17]. The authors focused on effective demand neglected his warning on paramount importance of conservation principle [17: 428]: "Goodwin employs the same definition, but one should notice that its meaning slightly changed, as profits now include the increment of inventories in case of excess supply and do not include sales of previously produced commodities in case of excess demand. This definition of profits does not cause serious problems as long as it is reasonable to expect that actual surpluses can be sold later. This is exactly what would happen in a non-explosive cyclical process with periods of excess surpluses being followed by times of excess demand and vice versa. Note, however, that we have neglected possible feedbacks from levels of inventories to changes in production."

The model in [17] is focusing on the role of effective demand in shortening the cycle against the original Goodwin's model (M-1 in the present paper). That model falls short of recognizing the decisive role of feedback loops containing the rate of capital accumulation in industrial cycles although Marx almost explicitly revealed these loops as this paper implicitly demonstrates on its first page [17: 423].

2.2. "Neoclassical hijacking" of M-1 in P-1

Table 2 lists main variables of P-1.

[4] G.W. Low [15] has shown that, when internal flows are conserved, a model of the multiplier-accelerator interaction does not produce short-term fluctuations, but does generate plausible, in his view, long-term cycles. The paper limits do not permit us discussing such cycles considered in [19, 23].

Table 2. Main variables in P-1

Variable	Expression
Net product	q
Fixed production assets	k
Capital-output ratio	$s = k/q$
Employment	l
Employment (in efficiency units)	$l_e = l e^{\alpha t}$, $\alpha > 0$
Output per worker	$a = q/l$
Labour force	$n = n_0 e^{\beta t}$, $\beta \geq 0$
Wage	w
Total wage	wl
Relative wage (unit value of labour power)	$u = w/a = wl/q$
Profit	$M = q - wl = (1-u)q$
Profit rate	$R = (1-u)/s$

In [4, 5] a CES production function is applied for determining net product

$$q = F(k, l_e) = c\left[\mu(m_\kappa k)^{-\delta} + (1-\mu)(m_e l_e)^{-\delta}\right]^{-1/\delta}, \tag{16}$$

where, according to "neoclassical" interpretation, μ is distribution parameter, $0 < \mu < 1$, c is efficiency parameter and δ is substitution parameter.

This function is linear and homogeneous, i.e., there are constant returns to scale by the standard definition. This definition overlooks scale effects maintained by specific feedback loops as next sections demonstrate. Function (16) has also a property of constant elasticity of substitution (CES) between *labour power* (in efficiency units) and *fixed production assets*[5] according to their alleged "marginal productivities" (F_{l_e} and F_k) under static conditions: $0 < \varepsilon = \dfrac{d\ln(k/l_e)}{d\ln\theta} = \dfrac{1}{1+\delta} < 1$ for $\theta = \dfrac{F_{l_e}}{F_k}$.

Parameters c, m_k and m_e help to harmonize units of measurement, each of the latter two equals 1, both are skipped for brevity. Function (16) allows considering variable capital-output ratio s, unlike M-1.

For $\varepsilon \to 1$ $(\delta \to 0)$ this function is transformed in the Cobb – Douglas function, $q = ck^{\mu} l_e^{1-\mu}$; for $\varepsilon \to 0$ $(\delta \to \infty)$ it becomes the Leontief technology, where $q = \mathrm{Min}(ck, cl_e)$, so capital-output ratio $s = \mathrm{const} = 1/c$. The first case represents in [4, 5] perfect factors' substitutability, the second case their perfect complementarity.

It is not taken into account in (16) that capital accumulation in not separable from technical progress embodied in fixed production assets, as N. Kaldor pointed with good reason [18]. The concept of "marginal productivities" defies the very essence of technology or of technical mode of production. Productivity of labour is a synthetic characteristic of the dual unity of concrete and abstract labour [1]. Productivity of capital is rigged notion that follows from denial of its essence as the specific production relation.

I have found out that production function (16) has the same CES in terms of k and l too: $F(k, l_e) \equiv \Phi(k, l)$ with $\varepsilon_l = \dfrac{d\ln(k/l)}{d\ln\theta_l} = \dfrac{1}{1+\delta}$ for $\theta_l = \dfrac{\Phi_l}{\Phi_k}$. This is because of the static character of this notion that requires treating $e^{\alpha t}$ for particular t as constant.

A modified Phillips equation defines the growth rate of wage

$$\hat{w} = f(v) + \rho\hat{a}. \tag{17}$$

A growth rate of wage is the sum of bargained $\hat{w}^m$ and stimulating $\hat{w}^b$ terms

$$\hat{w} = \hat{w}^m + \hat{w}^b, \tag{18}$$

[5] The papers [4, 5] recite uncritically the incorrect "neoclassical" notion of *capital-labour* substitution [8].

where the first is determined by employment ratio v as in a simplified Phillips equation

$$\hat{w}^m = f(v), \tag{19}$$

where $f'(v) > 0$, for $v \to 1$ $f(v) \to \infty$, and the second – by growth rate of output per worker according to the hypothesis on capitalists' "ability to pay" [5: 224]

$$\hat{w}^b = \rho\hat{a}, \tag{20}$$

where $0 \le \rho < 1/\varepsilon = 1 + \delta$.

Honestly, capitalists' ability to pay necessitates raising real wage no more than ability to drink whisky necessitates increased alcoholic consumption. F. van der Ploeg [5: 222, 224] did not sufficiently clear explain how capitalists are compelled to increase wage along with increases in output per worker. Specifically, a higher growth rate of output per worker hardly promotes workers' bargaining power similar to the employment ratio, as the thoughtful reader of [1], undoubtedly, understands.

A static problem of profitability maximization is considered for the modified Phillips equation

$$R = (1-u)/s = (q-wl)/k = (a-w)/(k/l) \to \max. \tag{21}$$

In result the growth in this model is wage-lead as profit and hence net product follow total wage with a lag of about one year duration in the simulation run below.

The offered optimal for capitalist class technological composition of capital – the relation between fixed production assets and labour power in efficiency units – is

$$k/l_e = [(1-\mu)(1-u)/(\mu u)]^{-1/\delta}, \tag{22}$$

correspondingly, optimal capital-output ratio is determined

$$s(u) = [\mu/(1-u)]^{1/\delta}/c. \tag{23}$$

Net change of fixed production assets (abstracting from investment delays) is accomplished by advancing a part of profit (surplus product) M:

$$\dot{k} = zM = z(1-u)q, \; 0 < z \le 1. \tag{24}$$

The growth rates of fixed capital is

$$\hat{k} = \frac{z(1-u)}{s(u)}. \tag{25}$$

The growth rates of capital intensity and output per worker were determined as

$$\widehat{k/l} = \alpha + \frac{1}{\delta}\left(\frac{\hat{u}}{1-u}\right), \tag{26}$$

$$\hat{a} = \alpha + \hat{u}/\delta. \tag{27}$$

For $0 \le \rho < 1$ (not $0 \le \rho \le 1 + \delta$ as in [5]), we uncover proportional control over the net change of relative wage:

$$\dot{u} = \frac{\delta}{(1-\rho)+\delta}[f(v) - (1-\rho)\alpha]u, \tag{28}$$

where adjustment coefficient $0 < \dfrac{\delta}{(1-\rho)+\delta} < 1$ (again for $0 \le \rho < 1$).

I've revealed combined proportional and derivative control over the net change of employment ratio:

$$\dot{v} = \left[\frac{z(1-u)}{s(u)} - \frac{1}{\delta}\frac{\hat{u}}{1-u} - (\alpha+\beta)\right]v \tag{29}$$

or

$$\dot{v} = z\left[\frac{1-u}{s(u)} - \frac{\alpha+\beta}{z}\right]v - \frac{1}{\delta}\frac{\hat{u}}{1-u}v. \tag{30}$$

Inside the square brackets in (30) there is the difference between the current profit rate and stationary one. Rate of capital accumulation z takes the role of the adjustment coefficient. The product $\frac{1}{\delta}\frac{1}{1-u}v$ determines a changeable strength of derivative control over v relying on $-\hat{u}$.

Equating the right parts (28) and (29) zero enables finding nontrivial stationary state

$$E_p = (u_p, v_p), \tag{31}$$

where $u_p = 1 - \left(\frac{\alpha+\beta}{cz}\right)^{\delta/(1+\delta)}\mu^{1/(1+\delta)}$ and $v_p = f^{-1}[(1-\rho)\alpha]$.

The stationary rate of growth of fixed production assets and net product is determined independently of rate of capital accumulation z as $\hat{k}_p = \hat{q}_p = \alpha+\beta$. The stationary employment ratio declines compared to M-1 if $1 > \rho > 0$.

Stationary rate of growth of output per worker, capital intensity and wage is defined as

$$\hat{a}_p = (k\hat{/}l)_a = \hat{w}_a = \alpha. \tag{32}$$

The stationary capital-output ratio and profit rate are specified as

$$s_p = [\mu/(1-u_p)]^{1/\delta}/c, \tag{33}$$

$$R_p = (1-u_p)/s_p = (\alpha+\beta)/z. \tag{34}$$

Ceteris paribus, the higher is rate of capital accumulation z, the higher are stationary relative wage u_p and capital-output ratio s_p and the lower is stationary profit rate R_p .

Figure 2 and Table 3 display a revealed causal loop structure of P-1.

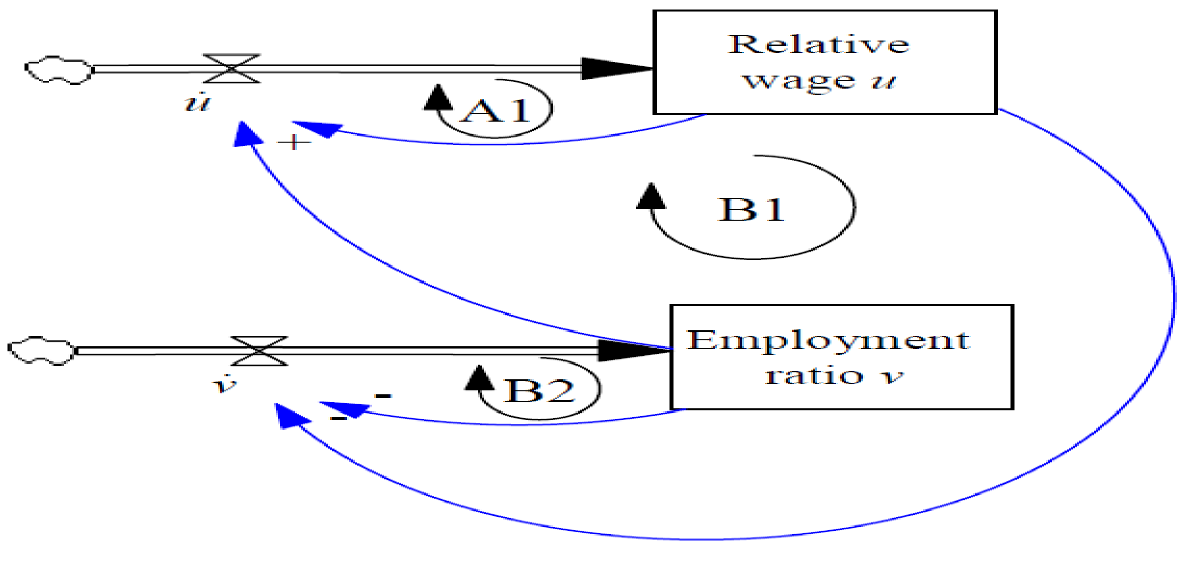

Figure 2 – A condensed causal structure of P-1 at stationary state E_p; total number of feedback loops – 3, among them: 1st order – 2 (1 – *negative*, 1 – *alternating*), 2nd order – 1 *negative*

First partial derivatives that are negative (–) and with alternating polarity (A) are explicitly shown. The other derivatives are positive. Negative feedback loop B1 is inherited from M-1, new negative feedback loop B2 is due to competition for jobs among workers. Besides these, P-1 includes single 1st order feedback loop with alternating polarity A1 for relative wage u inherited from M-1.

The P-1 intensive form is built into two-dimensional system of non-linear ODEs, local asymptotical stability (LAS) of its non-trivial stationary state is exposed in [5]. Our analysis founds out that proportional control over u and v, already present in M-1, is retained, whereas designed derivative control over v strengthens proportional control over v additionally in P-1.

Table 3. Three intensive feedback loops in P-1 at E_p

Loops descendant from M-1	New loop
A1 of length 1 Relative wage $u \xrightarrow{A}$ Net change of u	B2 of length 1 Employment ratio $v \xrightarrow{-}$ Net change of v
B1 of length 3 Relative wage $u \xrightarrow{-}$ Net change of v Employment ratio v Net change of u	

The CES production function (16) seemingly has a constant return to scale (according to the standard "neoclassical" definition with respect to the arguments k and l_e).

Still the standard "neoclassical" definition of economy of scale ignores feedback loops between growth rate of output per worker and other variables. Taking these feedback loops into account results in the deeper definition of economy of scale [19: 356–357].

Without going into details, *direct economy of scale* (*direct increasing return*) manifests itself in a positive partial derivative of growth rate of output per worker $\hat{a}$ with respect to employment ratio v or growth rate of employment ratio $\hat{v}$: $\frac{\partial \hat{a}}{\partial v} > 0$ (type I) or $\frac{\partial \hat{a}}{\partial \hat{v}} > 0$ (type II). *Roundabout economy of scale* (*roundabout increasing return*) manifests itself in a positive partial derivative of growth rate of output per worker with respect to employment ratio v or growth rate of employment ratio $\hat{v}$ intermediated by other variable or variables (x_i): $\frac{\partial \hat{a}}{\partial x_1}\frac{\partial x_1}{\partial v} > 0$ or $\frac{\partial \hat{a}}{\partial x_1}\cdots\frac{\partial x_i}{\partial v} > 0$, $\frac{\partial \hat{a}}{\partial x_1}\frac{\partial x_1}{\partial \hat{v}} > 0$ or $\frac{\partial \hat{a}}{\partial x_1}\cdots\frac{\partial x_i}{\partial \hat{v}} > 0$, $i \in \{2,\ldots,I\}$.

Economy of scale (increasing return) is *reinforcing* if a *positive* feedback loop connects the growth rate of output per worker with employment ratio and (or) its growth rate. Economy of scale (increasing return) is *weakening* if a *negative* feedback loop connects the growth rate of output per worker with employment ratio and (or) its growth rate.

For brevity and without loss of generality assume that ρ = 0. Then the growth rate of output per worker (27) is presented as

$$\hat{a} = \alpha + \hat{u}/\delta = \alpha + \frac{\hat{w} - \hat{a}}{\delta} =$$

$$= \frac{\alpha\delta + \hat{w}}{1+\delta} = \frac{1}{1+\delta}[\alpha\delta + f(v)]. \tag{35}$$

This presentation enables uncovering feedback loops containing $\hat{a}$ in P-1, particularly at the stationary state. Equation (35) reveals that P-1 is similar to the models applying the efficiency wage hypothesis stating that increases in wage promote output per worker. For example, a Goodwinian model with an efficiency wage mechanism from [20] reflects roundabout increasing returns $\frac{\partial \hat{a}}{\partial \hat{w}}\frac{\partial \hat{w}}{\partial v} > 0$ and $\frac{\partial \hat{a}}{\partial \hat{w}}\frac{\partial \hat{w}}{\partial v}\frac{\partial v}{\partial \dot{v}}\frac{\partial \dot{v}}{\partial \hat{v}} > 0$. In the case of Cobb – Douglas production function the efficiency wage hypothesis merges with constancy of relative wage since growth rates of wage and output per worker are virtually the same. The efficiency wage hypothesis does not recognize many aspects of the capitalist reality investigated in [11–13, 19, 23].

There is hidden presence of reinforcing roundabout economy of scale in P-1 containing CES production function $F(k, l_e)$, or CES production function $\Phi(k, l)$, although degree of homogeneity in both equivalent cases of (16) is constant and equals one according to the standard textbook definition.

Equating the "marginal rate of technical substitution" with the factor price ratio $\Phi_l / \Phi_k = w/R$ requires in P-1 shaky hidden assumption of "perfect" competition.[6] The latter is utter idealisation even for free compe-

[6] In the "neoclassical" conception, under "perfect" competition the "marginal rate of technical substitution" is equal to the relative unit costs of the inputs, so the slope of the isoquant at the chosen point equals the slope of the isocost curve. This equivalence is structurally fragile as section 3.2 demonstrates.

tition and is untrue for state-monopoly capitalism. I will return in section 3.2 to this assumption in [5].

Besides that the assumptions of constant returns to scale and widespread atomistic (“perfect”) competition are not compatible; it is necessary from the very beginning to assume a dynamic – not a static – substitution [22: 485–486, 488–492] and to overcome primary specification errors in CES production function stressed in [22: 496–498].

Table 4 reports on reinforcing roundabout economies of scale of types I and II in P-1 revealed neither in [5] nor in [4]. These three extensive positive feedback loops hardly posit any threat to LAS of E_p (31).

Table 4. Endogenous productivity growth and reinforcing roundabout economies of scale in P-1

No.	Order and polarity	Extensive feedback loop
1	1, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \widehat{k/l} \xrightarrow{-} \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$
2	1, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \rightarrow u \xrightarrow{-} \hat{k} \rightarrow \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$
3	2, +	$\hat{a} \xrightarrow{-} \hat{u} \rightarrow \dot{u} \rightarrow u \rightarrow s \xrightarrow{-} \hat{k} \rightarrow \hat{v} \rightarrow \dot{v} \rightarrow v \rightarrow \hat{w}$

Figure 3 displays dynamics in P-1 converging to stable node compared with self-sustained oscillations around a neutral centre in M-1. These models’ calibrating is purely illustrative.

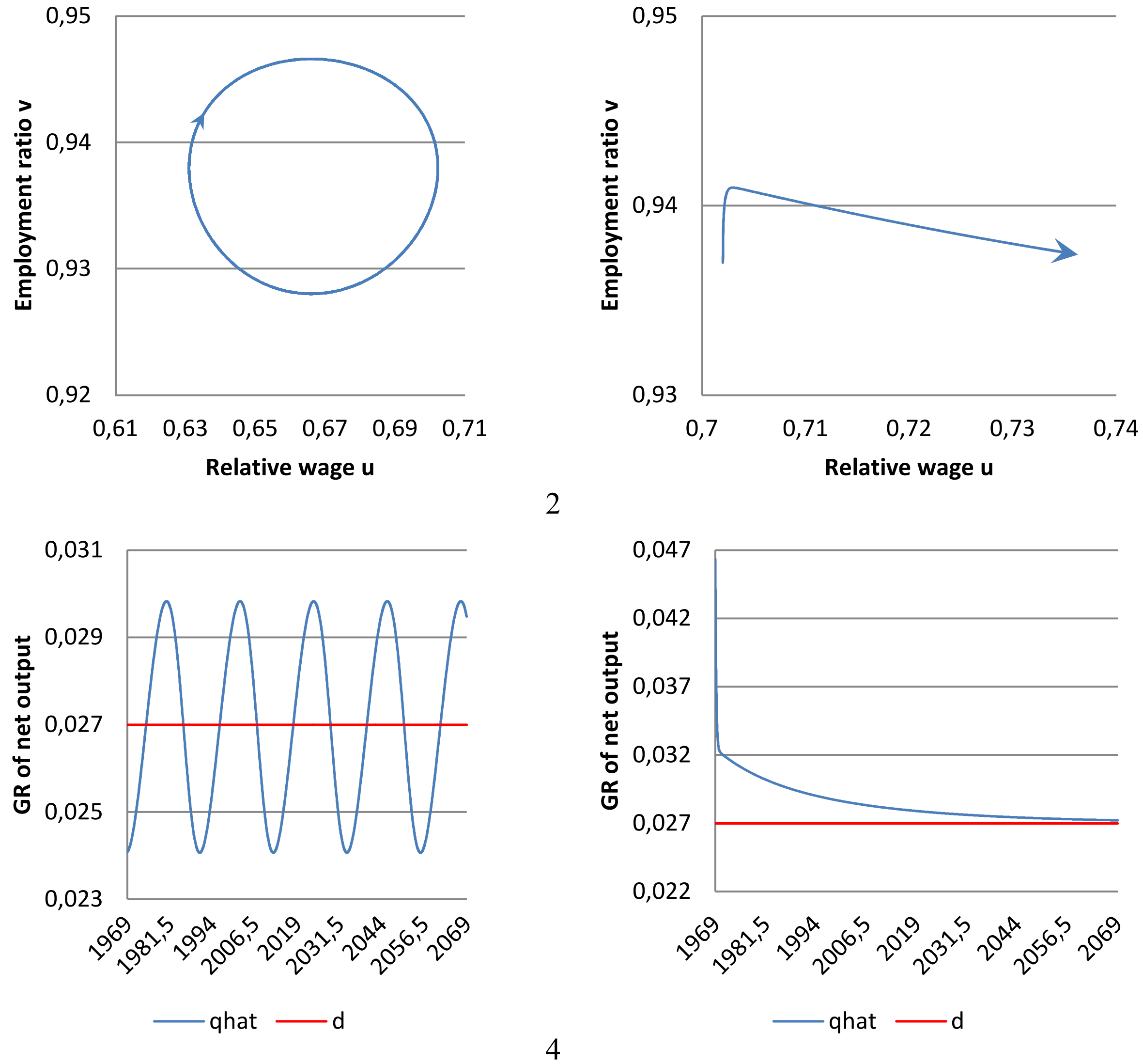

Figure 3 – A centre in M-1 (panels 1, 3) and stable node in P-1 (panels 2, 4), 1969–2069: employment ratio and relative wage (panels 1, 2); growth rate of net product and its stationary magnitude (panels 3, 4)

Equation (8) is specified as $f(v) = -g+r/(1-v)^2$. The initial state: $u_0 = 0.702$, $v_0 = 0.937$, $z = 0.156$; common parameters: $g = 0.04$, $\alpha = 0.012$, $\beta = 0.015$, $r = 0.0002$, $d = \alpha + \beta = 0.027$, additionally in M-1: stationary $u_G \approx 0.666$ and $v_G \approx 0.938$ and $f'(v)_G = 1.676$, initial $s_0 = 1.93$; additionally in P-1 $\delta = 0.33$, $\varepsilon = 0.752$, $\mu = 0.3$, $\rho = 0.1$, stationary $s_p = 1.512$, $u_p \approx 0.738 > u_G$ and $v_p \approx 0.937 < v_G$.

3. Check of P-1 structural stability in P-2 with additional scale effects

3. 1. Proportional and derivative control in P-2

As the present paper demonstrates, L. Aguiar-Conraria [4] has misperceived endogenous output per worker already present in P-1. Still his paper has extended the number of feedback loops involving growth rate of output per worker substantially that deserves readers' appraisal. Without these additional feedback loops in P-2 producing endogenous industrial cycles in subsequent Z-1 would be more difficult.

The former definition of employment (in efficiency units) l_e is in Table 2. It is redefined against P-1 taking into account direct scale effect

$$l_e = l e^{\alpha t} (k/k_0)^{\gamma}, \ \alpha > 0. \tag{36}$$

This newly defined l_e is the factor of CES production function (16).

A static problem of profitability maximization (21) is considered again for the same modified Phillips equation. In result the growth in this model is also wage-lead as profit and hence net product follow total wage with a lag measured by few months that is shorter than in P-1.

New equations for the growth rates of capital intensity and output per worker extend (26) and (27):

$$\widehat{k/l} = \alpha + \frac{1}{\delta}\left(\frac{\hat{u}}{1-u}\right) + \gamma \hat{k}, \tag{37}$$

$$\hat{a} = \alpha + \hat{u}/\delta + \gamma \hat{k}. \tag{38}$$

Intensive form of P-2 follows is a system of two ODEs that generalize (28) and (29)

$$\dot{u} = \{f(v) - (1-\rho)[\alpha + \gamma \frac{z(1-u)}{s(u)}]\} \frac{\delta u}{(1-\rho)+\delta}. \tag{39}$$

$$\dot{v} = [(1-\gamma)\frac{z(1-u)}{s(u)} - \frac{1}{\delta}\frac{\hat{u}}{1-u} - (\alpha+\beta)]v. \tag{40}$$

Workers' cohesion in the fight for increased relative wage is the stronger the more is $\partial \dot{u}/\partial u$ above zero. On the one hand, the intensity of their competition for jobs is the stronger the deeper is $\partial \dot{v}/\partial v$ below zero. In equation (39), $f(v) \to \infty$ for $v \to 1$ again.

Proportional control over the net change of relative wage as in P-1 is strengthened by derivative control over it in (41). Combined proportional and derivative control over the net change of employment ratio already present in P-1 is mostly retained in (42) although the adjustment coefficient is $(1-\gamma)z$ now instead of z in P-1. These properties are seen in the corresponding equations:

$$\dot{u} = \frac{\delta u}{(1-\rho)+\delta}[f(v) - (1-\rho)\alpha] - (1-\rho)\gamma \frac{\delta u}{(1-\rho)+\delta}\hat{k}, \tag{41}$$

$$\dot{v} = (1-\gamma)z[\frac{(1-u)}{s(u)} - \frac{d}{z}]v - \frac{1}{\delta}\frac{\hat{u}}{1-u}v. \tag{42}$$

Thus the scale effect serves as additional fix in P-2 and does not endanger local asymptotic stability of a new stationary state that will be soon defined.

Instead of assumption $z = 1$, now this parameter belongs to semi-interval defined as

$$0 < z_{\inf} = dc^{\delta}/\mu < z = const \le 1. \tag{43}$$

Here the left boundary follows from requirement that a stationary relative wage is positive and stationary capital-output ratio is higher than 1 (see Proposition 2 below). The upper boundary is a bit too high as it permits at the extreme (z = 1) zero private capitalists' consumption. As $z = const \le 1$ enables comparisons of the modified models with the original models which used $z = const = 1$ the upper boundary will remain unchanged.

Equating the right parts (39) and (40) zero makes possible finding nontrivial stationary state

$$E_a = (u_a, v_a). \tag{44}$$

where $u_a = 1 - \left(\frac{d\mu^{1/\delta}}{cz}\right)^{\delta/(1+\delta)}$ and $v_a = f^{-1}[(1-\rho)(\alpha+\gamma\beta)/(1-\gamma)]$.

For this stationary state, the rate of growth of output per worker, capital intensity and wage is defined

$$\hat{a}_a = (\widehat{k/l})_a = \hat{w}_a = (\alpha+\gamma\beta)/(1-\gamma), \tag{45}$$

as well as the stationary rate of growth of fixed production assets and net product is determined

$$\hat{k}_a = \hat{q}_a = \hat{a}_a + \beta = (\alpha+\beta)/(1-\gamma) = d. \tag{46}$$

The stationary capital-output ratio and profit rate are specified as

$$s_a = [\mu/(1-u_a)]^{1/\delta}/c, \tag{47}$$

$$(1-u_a)/s_a = d/z. \tag{48}$$

There is stationary employment ratio – stationary relative wage trade-off in P-2: the higher γ, the higher is the first and the lower is the second.

In the "neoclassical" conception, the stationary relative wage u_a, being the higher, ceteris paribus, the higher is δ, aspires to supremum when $\delta\to\infty$ (Leontief technology with factors complementarity): $\sup(u_a) = 1 - d/(cz)$; stationary relative wage u_a, being the lower, the lower is δ, aspires to infimum when $\delta\to 0$ (Cobb – Douglas production function with perfect factors substitutabity): $\inf(u_a) = 1 - \mu$.

Increase in stationary rate of economic growth d affects the relative remuneration of u_a negatively; $u_a < 1$ is true only if $d > 0$ [5: 225].

Figure 4 and Table 5 exhibit intensive causal structure of P-2.

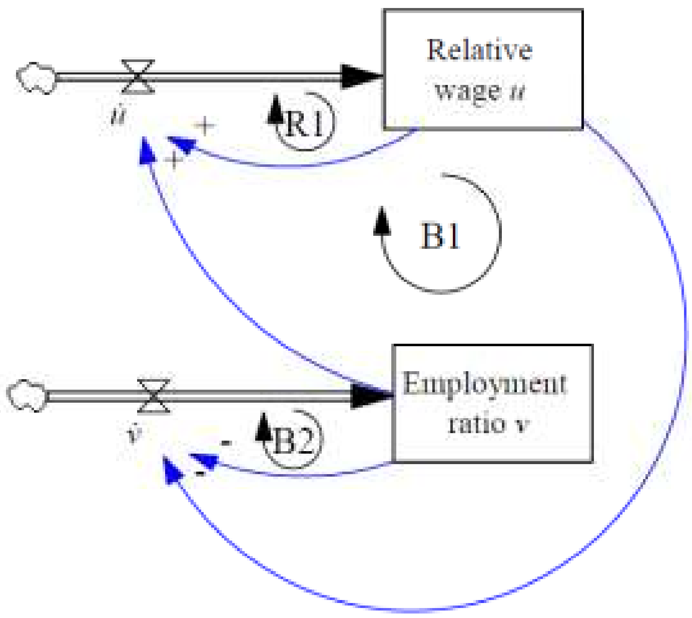

Figure 4 – A condensed causal structure of P-2 at stationary state E_a; total number of feedback loops – 3, among them: 1st order – 2 (1 – *positive*, 1 – *negative*), 2nd order – 1 *negative*

Table 5. Three intensive feedback loops in P-2 at the stationary state E_a

Loops descendant from P-1	New loop R1 of length 1
B2 of length 1 Employment ratio $v \xrightarrow{-}$ Net change of v	Relative wage u Net change of u
B1 of length 3 Relative wage $u \xrightarrow{-}$ Net change of v Employment ratio v Net change of u	

We see that this model inherits two negative feedback loops from P-1 and includes the new positive feedback loop that is destabilizing and results from workers' joint struggle for higher relative wage.

3.2. Conceptual weakness of P-2 rooted in "neoclassical" beliefs

The positive dependence of $\hat{a}$ on $\hat{u}$ in (27) and (38) did not receive clear explanation in [4, 5]. For filling this gap this research digs deeper. For brevity and without loss of generality assume again that ρ = 0. Then the growth rate of output per worker (38) is presented for P-2 retaining the efficiency wage hypothesis as extension of (35) in P-1

$$\hat{a} = \frac{\alpha\delta + \hat{w} + \delta\gamma\hat{k}}{1+\delta} = \frac{1}{1+\delta}[\alpha\delta + f(v) + \delta\gamma\hat{k}]. \quad (49)$$

The efficiency wage hypothesis is clearly relaxed in (49) in relation to (35) in P-1.

The three feedback loops reflecting endogenous productivity growth and scale effects in P-1 (Table 4) are also present in P-2.

Table 6 reports on four additional extensive feedback loops positing no serious threat to LAS of E_a (44). L. Aguiar-Conraria [4] does not recognize them. Two latter reflect roundabout weakening and stabilising economies of scale of types I and II in P-2. An analogue of degree of homogeneity for VES production function $\Phi(k, l)$ is generally not constant and exceeds 1.

Table 6. Endogenous productivity growth and scale effects in P-2

No.	Order and polarity	Extensive feedback loop involving growth rate of output per worker
4	1, +	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \xrightarrow{-} \hat{k}$
5	1, +	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \to s \xrightarrow{-} \hat{k}$
6	2, -	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \xrightarrow{-} \hat{k} \to \widehat{k/l} \xrightarrow{-} \hat{v} \to \dot{v} \to v \to \hat{w}$
7	2, -	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \to u \to s \xrightarrow{-} \hat{k} \to \widehat{k/l} \xrightarrow{-} \hat{v} \to \dot{v} \to v \to \hat{w}$

Fixed capital formation is beneficial for growth, according to (16), (38) and (49). However, the latter two equations express the direct scale effect not quite correctly. Refinement requires that the growth rate of output per worker is linked to the growth rate of employment ratio instead of the growth rate of fixed production assets [19]. In our opinion, $\hat{a}$ is in reality directly dependent on the growth rate of capital intensity besides its dependence on the growth rate of employment ratio.

The scale effects in (16), (38) and (49) require 0 < γ < 1 as assumed in P-2 contrasting with P-1, where γ = 0 implicitly. On the other hand, ρ = 0 in P-2 unlike P-1.

The principal concern will be with the property of homogeneity of the first degree of CES production function (16) in relation to l_e and k. This property holds in relation to l and k only if γ = 0 as in P-1.

The scale effect, intended by L. Aguiar-Conraria, violates the distribution of net product between labour and capital according to their "marginal productivities" Φ_l и Φ_k in P-2 for $\gamma > 0$.

Production function (16) is to be easily expanded in true final terms of l and k instead of l_e and k, where l_e is intermediate variable for l. It becomes clear that the expanded VES production function $\Phi(k, l)$ is not generally homogenous in terms of l and k, therefore the Euler theorem for homogenous functions cannot be applied, except the Cobb – Douglas special case with a degree of homogeneity expressed as $1 + \gamma(1-\mu)$.

Strictly speaking, the profit maximization condition equates the value of the "marginal product of labour" with the wage rate under "perfect" competition that is very strong idealisation even of free competition. Neither [5] nor [4] is explicit on this crucial "neoclassical" assumption, their authors have simply avoided discussion of this.

Seemingly, the Phillips equation is maintained by assumption of bargaining strength of the workers' movement [5: 222]. Thus, at least, a countervailing power of trade-unions against monopoly and firms' monopsony on the labour market is recognizable (cf. [16: xvii]). This recognition is not followed by investigation of monopoly capitalism in depth. Therefore applying the profit maximization condition developed for "perfect" competition creates unresolved logical inconsistencies in P-1 and P-2. Yet the problem is even more severe – it cannot be removed by simple refinement as the following proposition establishes.

Proposition 1. The logically contradictory principle of distribution of national income (net product) is introduced without any theoretical justification in P-2: on the one hand, wage is determined there by labour "marginal product" $w = \Phi_l$, and $lw = l\Phi_l$, on the other hand, profit rate is lower than the "marginal product" of capital $R = M/k = (1-u)/s = F_k < \Phi_k$ if $1 > \gamma > 0$.

Corollary 1. Profit is lower than imputed profit: $M = (1-u)q = q - \Phi_l l < \Phi_k k$.

Corollary 2. Net product is lower than imputed total income: $q < \Phi_l l + \Phi_k k$.

Corollary 3. The "factor price ratio" exceeds "marginal rate of technical substitution": $w/R > \Phi_l / \Phi_k$. So the "marginal rate of technical substitution" is not equal to the "relative unit costs" of the inputs; in other words, the slope of the isoquant at the chosen point is not any more equal to the slope of the isocost curve.

An easy algebraic proof of this *Proposition* and its *Corollaries* is omitted. Proofs are skipped in similar cases below as well.

The "neoclassical" marginal productivity principle of income distribution is broken in [4] in silence. In fact, the relative wage u expresses the value of labour power; the profit of capitalists M is a transformed form of surplus value S created by the working class: $S = (1 - u)L = M/a$.

Proposition 2: (a) from the requirement $u_a > 0$ the first restriction follows $1 \geq z > d\mu^{1/\delta}/c > 0$; (b) from the requirement $s_a > 1$ the second (stronger) restriction follows $1 \geq z > \inf(z) = dc^{\delta}/\mu$.

Proposition 3: (a) stationary capital-output ratio s_a, being the lower, the higher is δ, ceteris paribus, aspires to infimum when $\delta \to \infty$ (Leontief technology with factors complementarity): $\inf(s_a) = 1$; (b) stationary capital-output ratio s_a, being the higher, the lower is δ, aspires to supremum when $\delta \to 0$ (Cobb – Douglas production function with perfect factors substitutabity): $\sup(s_a) = \mu z/d$.

Proposition 4. (a) Stationary relative wage u_a and stationary capital-output ratio s_a, being the higher, the higher is rate of accumulation z, achieve maximum when $z = 1$; (b) stationary employment ratio v_a is independent of z; (c) stationary growth rates $\hat{a}_a = (\widehat{k/l})_a = \hat{w}_a$, $\hat{k}_a = \hat{q}_a$ do not depend on z too; (d) stationary profit rate d/z, being the lower, the higher is rate of accumulation z, achieves minimum when $z = 1$. Therefore real capitalists eschew the well-known "golden rule" of accumulation that requires from them $z = 1$ against their material interests (see for details [11]).

Proposition 5. The stronger is the solidarity of workers in struggle for the relative wage, the higher is δ, and therefore strengthening this solidarity is a means of enhancing stationary relative wage (value of labour power) u_a. This increase has no influence on a stationary rate of return $(1 - u_a)/s_a$, as stationary capital-output ratio s_a declines along with accrual in δ.

Proposition 6. For $0 < \gamma < 1$: (a) LAS of hyperbolic E_a in system of (41) and (42) takes place according to Routh – Hurwitz criterion, if $Trace(J_a) < 0$ [4: 522]; in particular, (b) E_a is locally stable node for $0 < \delta \le \delta_1$, (c) E_a is locally stable focus for $\delta_1 < \delta \le \delta_2$, here δ_1 is such that $[Trace(J_a)/2]^2 = |J_a|$, δ_2 is such that $Trace(J_a) = 0$.

Corollary. LAS of hyperbolic E_a is guaranteed as far as

$$\gamma < \frac{1}{(1-\rho)(1+\delta)} \frac{f'(v_a)}{d} \frac{v_a}{u_a}. \tag{50}$$

Notice that d, v_a and u_a depend on γ (cf. [4: 522]).

LAS of hyperbolic E_a, in particular, is true for $\gamma = 0$, $z = 1$ and $\delta < \infty$. Similarly, for $\gamma = 0$ in P-1, LAS of E_p is guaranteed. Stationary state E_a (44) becomes neutral centre instead of being hyperbolic similar to M-1 if $\gamma = 0$ as in P-1 and there is no workers' competition for jobs when $\delta \to \infty$, $s_a \to 1$, $Trace(J_a) \to 0$.

According to [4: 524], "[the] stabilizing effect of introducing some flexibility in the production function is much stronger than the destabilizing effect of endogenous productivity growth. Only when the production function is extremely close to a Leontief technology does the system generate perpetual (and explosive) oscillations."

Such oscillations with period of 24–45 years require unrealistically low $s_a \approx 1$ for plausible z. If $z = 1$, this model, similar to M-1 and P-1, can produce converging fluctuations with period of about 10 years. Thus for keeping them in life exogenous shocks are necessary as in so-called real business cycles. Sticking to scientific truth, those cycles "of the Frisch type" are not real – they are artificial and ill-defined [21: 227–233].

4. Model Z-1 of industrial cycles as capital accumulation cycles

The reader remembers that the apologetic "marginal productivity" principle underlies P-1. Strictly speaking it must assume "perfect" competition in the factor market which is unrealistic especially under state-monopoly capitalism. The attempt to bring P-1 closer to reality through modification P-2 in [4] has undermined this principle. It follows from above *Proposition* 1 that the "neoclassical" beliefs have moved P-1 and P-2 to dead end. "Is there no exit?" – "No, it is on the opposite side!"

The model that follows is to a large extent liberated from these erroneous beliefs, in particular, from the "marginal productivity" principle that still remains as rudiment. The labour theory of commodity and surplus value is applied as the foundation for Marxian interpretation and for reasonable, still uncompleted, re-shaping of the preceding models.

4.1. The intensive form of Z-1 with endogenous rate of capital accumulation

The following K. Marx fragment tells how rate of accumulation is determined [1: 634]: "If the quantity of unpaid labour supplied by the working class, and accumulated by the capitalist class, increases so rapidly that its conversion into capital requires an extraordinary addition of paid labour, then wages rise, and, all other circumstances remaining equal, the unpaid labour diminishes in proportion. But as soon as this diminution touches the point at which the surplus labour that nourishes capital is no longer supplied in normal quantity, a reaction sets in: a smaller part of revenue is capitalised, accumulation lags, and the movement of rise in wages receives a check. The rise of wages therefore is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale. The law of capitalistic accumulation, metamorphosed by economists into pretended law of Nature, in reality merely states that the very nature of accumulation excludes every diminution in the degree of exploitation of labour, and every rise in the price of labour, which could seriously imperil the continual reproduction, on an ever-enlarging scale, of the capitalistic relation."

The negative feedback of the 3rd order containing the rate of accumulation z, employment ratio v and labour value u, implicitly expressed by K. Marx [1: 634], is presented on Figure 5.

J. Robinson, as [17: 423] reminded us, commented in some detail on this part of Marx's theory. She argued [16: 84–85] that Marx was mistaken in presenting his model as an explanation of the business cycle: "This cycle Marx identifies with the decennial trade cycle. This identification is an error…There may be in reality a cycle of the type which Marx analyses. But if so, it must be of a much longer period than the decennial trade cycle...”

After demonstrating in preceding research [19, 23] based on K. Marx’s theory that a cycle with much longer period of about three to five decades is quite possible indeed, the later investigation [13] does not agree with J. Robinson’s downgrading of the decennial trade cycle identified by K. Marx. The present paper reinforces support for the Marx industrial cycle.

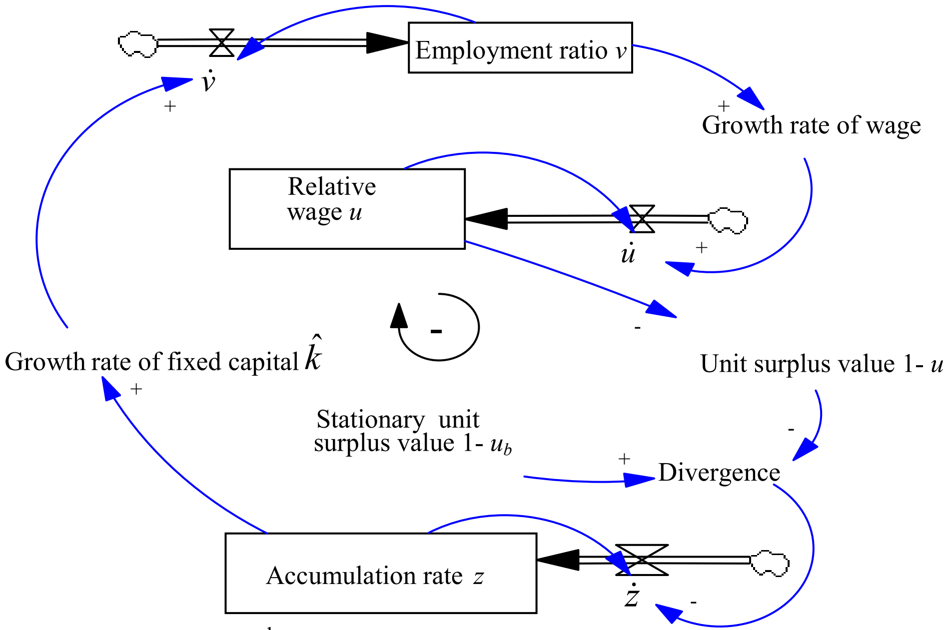

Figure 5 – The negative extensive 3rd order feedback loop for rate of accumulation, employment ratio and value of labour power implicitly expressed in K. Marx [1]

The papers [11–13] have turned rate of accumulation z in a new base (phase) variable. The following soon equation (51) takes into account, first, in agreement with the views of K. Marx above, that net change of the share of investment in surplus product has an opposite sign in response to relative wage gains:

$$\dot{z} = -b\frac{\dot{u}}{1-u}z(Z-z) + p(z_b - z), \tag{51}$$

where $b \geq 0,\ p > 0,\ z_b < z_0 \leq 1 < Z$.

This equation, second, reflects objective interest of capitalists in the long-term increase of the rate of profit; restrictions $p > 0$ and $z_b < z_0$ serve a long run increasing profit rate. Third, the product $z(Z - z)$ reflects logistical dependence of $\dot{z}$ on z that bounds trajectories in the phase space while a magnitude of Z codetermines amplitude of fluctuations.

Z-1 extends the equations of P-2 by (51). Z-1 includes the other equations of P-2 without z unchanged. Z-1 also includes the other equations of P-2 with z – yet now this is phase (level) variable instead of being constant in P-2 (as in M-1 and in P-1 above).

The same static problem of profitability maximization (21) is considered for the given modified Phillips equation (17) again in Z-1. Although in Z-1, as in P-2, “marginal productivity of capital” exceeds the profit rate, the rudiment “neoclassical” equivalence of “marginal productivity of labour” and wage remains in agreement with Proposition 1 in section 3.2.

Now the intensive form of Z-1 including the renewed investment function consists of the following three non-linear ODEs: equations (39), (40) and original (51) for rate of capital accumulation z. Accordingly, the interpretation has changed for equations (39) and (40).

The system (39), (40) and (51) has stationary state

$$E_b = (u_b, v_b, z_b), \tag{52}$$

where u_b and v_b are the same as u_a and v_a in (31) for $z = z_b$, $0 < z_b = z_{goal} < 1$, restrictions on z_b also remain the same as in Proposition 2. Equations (32)–(34) define the stationary auxiliary magnitudes for Z-1 as well.

Figure 6 and Table 7 reflect a condensed causal loop structure of Z-1 near stationary state E_b (52).

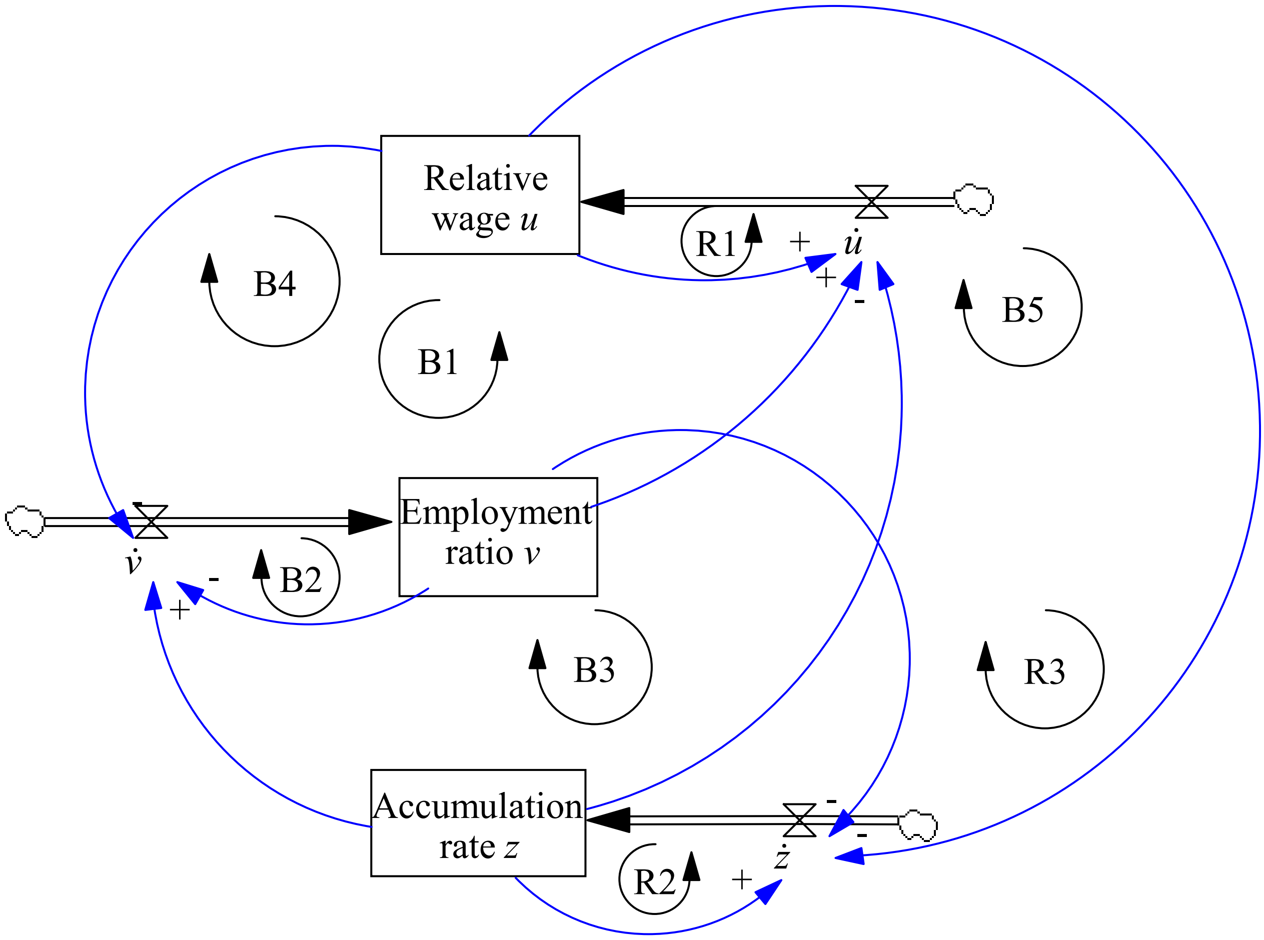

Figure 6 – A condensed causal loop structure of Z-1 at E_b; total number of feedback loops – 8, among them: 1st order – 3 (1 – negative, 2 – *positive*), 2nd order – 3 (2 – negative, 1 – *positive*), 3rd order – 2 (2 – negative)

Table 7. The intensive feedback loops in Z-1 at stationary state E_b

Quantity	Order	Positive feedback loop	Negative feedback loop
3	1st	R1 of length 1 $u \to \dot u$ R2 of length 1 $z \to \dot z$	B2 of length 1 $v \xrightarrow{-} \dot v$
3	2nd	R3 of length 3 $u \xrightarrow{-} \dot z \to z \xrightarrow{-} \dot u$	B1 of length 3 $u \xrightarrow{-} \dot v \to v \to \dot u$ B3 of length 3 $v \xrightarrow{-} \dot z \to z \to \dot v$
2	3rd		B4 of length 5 $u \xrightarrow{-} \dot z \to z \to \dot v \to v \to \dot u$ B5 of length 5 $v \xrightarrow{-} \dot z \to z \xrightarrow{-} \dot u \to u \xrightarrow{-} \dot v$

Note. M. Happach has perfectly labelled R2 and R3 as greed feedback loops – very catching indeed! This valuable insight is gratefully acknowledged again here as at my poster presentation in Delft on July 18, 2016.

There are three feedback loops inherited from M-1, P-1 and P-2 (B1, B2 and R1) as well as five new ones (B3, B4, B5, R2 and R3). The three feedback loops from P-1 (Table 4) and the four feedback loops from P-2 (Table 6) are characteristics of the vicinity of stationary state E_b in Z-1. Table 8 reports on three additional extensive feedback loops that together potentially threaten LAS of E_b. The second of them (No. 10) reflects reinforcing roundabout economy of scale of types I and II, the third (No. 11) – weakening and stabilising roundabout economy of scale of types I and II. Neither F. van der Ploeg [5] nor L. Aguiar-Conraria [4] recognized these loops. The effects of scale are clearly strengthened in Z-1 with respect to P-2 and P-1.

Table 8. Endogenous productivity growth and additional scale effects in Z-1 at stationary state E_b

No.	Order and polarity	Extensive feedback loop
9	1, +	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \xrightarrow{-} \hat{z} \to \dot{z} \to z \to \hat{k}$
10	2, +	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \xrightarrow{-} \hat{z} \to \dot{z} \to z \to \hat{k} \to \hat{v} \to \dot{v} \to v \to \hat{w}$
11	2, –	$\hat{a} \xrightarrow{-} \hat{u} \to \dot{u} \xrightarrow{-} \hat{z} \to \dot{z} \to z \to \hat{k} \to \widehat{k/l} \xrightarrow{-} \hat{v} \to \dot{v} \to v \to \hat{w}$

Capitalists' investment cooperation is the stronger the more $\partial \dot{z} / \partial z$ exceeds zero, and the intense competition in this field involves $\partial \dot{z} / \partial z < 0$. The first is destabilizing, the second – stabilizing. Notice that Propositions 1–6 and their Corollaries remain untouched.

Proposition 7. The dynamics of system (39), (40) and (51) linearized in the neighbourhood of its hyperbolic stationary state E_b (52) are LAS provided that $0 \le b < b_0 < b_2 < \infty$. Then stationary state E_b is also LAS in the non-linear system (39), (40) and (51). Stationary state E_b is *not* stable for $b \ge b_0$ in the linearized system (39), (40) and (51).

Corollary. (a) If the stationary state E_b (52) is LAS, it saves this property if b becomes lower than its initial magnitude b_i. If the stationary state E_b is not, it gets this property if b becomes sufficiently lower than its initial magnitude b_i. If the stationary state E_b is LAS, it loses this property if b becomes sufficiently higher than its initial magnitude b_i. (b) The stationary state E_b is LAS for $b = 0$ and $p > 0$. (c) The stationary state E_b is LAS for the two-dimensional case of Z-1 with $b = 0$, $p = 0$, $z = \text{const}$ as in M-1, P-1 and P-2.

In our particular simulation run stationary state E_b is not stable in linearized Z-1: $a_0 \approx 0.0028 > 0$, $a_1 \approx 0.8932 > 0$, $a_2 \approx 0.0032 > 0$, $a_1a_2 - a_0 \approx 0.0000$; correspondingly, $b_1 = -37.7085 < b_3 = -37.6209 < b_0 = 54.3987 < b_2 = 54.4863$. Still stationary state E_b is stable in nonlinear Z-1 up to $b_{critical} = b_0 + 3 = 57.3987$.

4.2. Super-critical Andronov – Hopf bifurcation and self-sustained industrial cycles

Proposition 8. The Andronov – Hopf bifurcation does take place in the system (39), (40) and (51) in a local vicinity of E_b (52) at $b = b_0$ defined by equation (75).

It has been proved that E_b (52) is locally asymptotically stable for $b < b_0$ and that the Andronov – Hopf bifurcation does take place in the system (39), (40) and (51) at $b = b_0$. According to simulations, a supercritical bifurcation occurs. The period of oscillations near E_b is about $2\pi/\sqrt{a_1(b_0)} \approx 6.648$ (years).

For $\gamma = 0.75$ and $b = b_{critical} = 57.3987 > b_0 = 54.3987$, there is a transition to a limit cycle vicinity (up to years 2200–2230) from the initial phase vector x for 1958.

Consider the conditions in which experimental limit cycle stands idealization of industrial cycle. In (8) function $f(v) = -g+r/(1-v)^2$ is used. The roughly plausible values prompted by [4, 5] have served in simulation runs: $\alpha = 0.005$, $\beta = 0$, $\gamma = 0.75$, $\delta = 1$, $\varepsilon = 0.5$, $\mu = 0.3$, $\rho = 0$, $p = 0.2$, $c = 1$, $g = 0.04$, $r = 0.001$, $d = 0.02$, $s_a = s_b = 1.342$, $u_a = u_b = 0.776$, $v_a = v_b = 0.871$, $z = z_b = 0.12$, $Z = 1.5$, $u_0 = 0.83$, $s_0 = 1.764$, $v_0 = 0.9$, $z_0 = 0.267$, $(1 - u_0)/s_0 = 0.0964$.

For $b_{critical} = b_0 + 3 = 57.3987$ a supercritical Andronov – Hopf bifurcation happens giving birth to a limit cycle that depends on the initial vector x_0. In addition, for those same parameters and initial conditions, limit cycles in the economic subspace ($z \le 1$) arise even for $b_{critical} = b_0 + 7.875 = 62.274$. Starting year in numerical experiments is denoted for certainty as 1958. To about 2200 and later on movement has become regularly established near the limit cycle that cannot be reproduced with absolute precision.

Net product reaches its local maximum on the completion of the boom with the onset of the crisis. Ending the fall of net product q expresses completion of crisis, whereas achieving pre-crisis peak completes recovery. Depression is defined as phase starting at the end of the crisis and ending before recovery, when capital-output ratio s is (locally) maximal. Cycle phases are reflected in Table 9.

Table 9. Phases of cycle of capital accumulation in the vicinity of the limit cycle, 2221–2227.75

Phase of industrial cycle	Phase period	Quantity of quarters	Beginning	End
Crisis – the 1st phase of the cycle	2221.25–2222	4	1	4
Depression	2222.25–2222.75	3	5	7
Recovery	2223–2223.25	2	8	9
Boom	2223.5–2227.75	18	10	27
Complete cycle	2221.25–2227.75	27 quarters or 6.75 years		

The growth rate of net product ranges from –1 to 6 (%/y.), rate of accumulation – from 0.054 to 0.471 (the stationary 0.12 inside this interval), employment ratio – from 0.857 to 0.891 (the stationary 0.871 inside this interval), relative wage – from 0.819 to 0.825 (above the stationary 0.776), capital-output ratio – from 1.661 to 1.712 (above the stationary 1.341), profit rate – from 0.102 to 0.109 (below the stationary 0.167), surplus value – from 0.141 to 0.151, labour value of net investment – from 0.008 to 0.071.

During the allied industrial cycles with a period of 6.75 y. in preparing crisis the chronicle sequence is the following (Table 10): (max 2220) declining rate of accumulation → (max 2220.25) falling rate of profit, (max 2220.25) falling surplus value, (max 2220.25) declining employment ratio → (max 2220.75) falling profit → (max 2221) fall of net product. The exit from the crisis involves: (min 2222) increasing net output → (min 2222.25) increasing profit and rate of accumulation → (min 2222.5–2222.75) increasing profit → (min 2222.75–2223) increasing surplus-value → (min 2223–2223.25) increasing employment ratio.

The cyclical dynamics in this model are not strictly wage-lead as in P-1 and P-2: profit falls from the local high about half a year before total wage, still the latter resumes growth about half a year before profit. Net product follows total wage with time interval of about one quarter out of the bottom, whereas production declines few weeks earlier than total wage. In contrast to the above Marx' views [1], reduction of wage w does not occur, however, in agreement with his views, employed and unemployed workers' consumption per capita wv declines on the crisis phase and reaches its maximum in the late phase of boom.

Table 10. Phases of cycle of capital accumulation in the vicinity of the limit cycle, 2221–2227.75

	Boom (end)	Crisis			Depression		
Year	2221	2221.25	2221.75	2222	2222.25	2222.5	2222.75
Quarter		1	3	4	5	6	7
q	max 1			min 1			
$\hat{v}$	min						
v							
s							max
u							max
Profit rate R						min	min
S							min
M					min 1		
w steady growth	min						
wl	max 1			min 1			max 1
wv	max 1			min 1			max 1
z					min		
$\dot{k}/a$					min		

Table 10 (continued)

	Recovery		Boom							
Year	2223	2223.25	2223.5	2224.5	2226.25	2226.75	2227	2227.25	2227.5	2227.75 (end)
Quarter	8	9	10	14	21	23	24	25	26	27
q		max 1								max 2
$\hat{v}$	0	0			max					min
v	min	min					max			
s							min			
u							min			
Profit rate R							max			
S	min						max	max		
M				max 1 2220.5					max 2	
w										max
wl										max 2
wv										max 2
z						max				
$\dot{k}/a$						max				

Note. A critical magnitude of parameter b for the Andronov – Hopf bifurcation at E_b (58) is $b_{critical}$ = 57.3987.

This brief exposition will be continued in next subsection.

4.3. Dual nature of capital as the driver and barrier of capitalist production

Excess of capital arises from the same causes as those which call forth unemployment – complementary phenomena, footing at the opposite poles.

Positive declining profit rate $R = \frac{1-u}{s}$ ($\hat{R} < 0$) is the indicator for relative excess of capital:

$$\hat{R} = -\frac{\dot{u}}{1-u} - \hat{s} < 0. \tag{53}$$

In P-1, P-2 and Z-1, $\dot{u}$ and $\hat{s}$ have always the same sign: sign($\dot{u}$) = sign($\hat{s}$); therefore profit rate R always declines when relative wage u and synchronically capital-output ratio s rise in these models. So minimum of R happens synchronically with local maximums of u and s, and vice versa for cyclical dynamics in Z-1.

A deeper analysis distinguishes two forms of absolute excess of capital.

1) If the fall in the rate of profit is not compensated through the mass of profit, when the increased capital produced just as much, or even less, profit than it did before its increase:

$$\hat{M} = \hat{S} + \hat{a} = \hat{q} - \frac{\dot{u}}{1-u} \le 0 \tag{54}$$

therefore

$$\hat{S} \le -\hat{a}, \tag{55}$$

or

$$\hat{q} \le \frac{\dot{u}}{1-u}\ . \tag{56}$$

2) Similarly, if the fall in the profit share (unit surplus value) is not compensated through the mass of surplus labour, when the increased capital produced just as much, or even less, surplus-value than it did before its increase:

$$\hat{S} \le 0 \equiv \hat{l} - \frac{\dot{u}}{1-u} \le 0 \tag{57}$$

or

$$\hat{l} \le \frac{\dot{u}}{1-u}. \tag{58}$$

It develops for constant labour force ($n = const$) in two identical requirements $\hat{S} \le 0 \equiv \hat{v} - \frac{\dot{u}}{1-u} \le 0$.

Absolute over-accumulation of type I is sufficient for relative over-accumulation of capital. Indeed, $\hat{M} = \hat{k} + \hat{R} < 0$ therefore for $\hat{k} > 0$ $\hat{R} < -\hat{k} < 0 \Rightarrow \hat{R} < 0$.

Let the relative over-accumulation occurs: for $\hat{k} > 0$, $R > 0$ and $\hat{R} < 0$.

Relative over-accumulation of capital is not necessarily accompanied by absolute capital over-accumulation of type I, or the former is not a sufficient condition for the latter: for $\hat{R} < 0$, $\hat{M} > 0$ if $\hat{k} + \hat{R} > 0$ therefore for $\hat{k} > -\hat{R} > 0$.

Absolute over-accumulation of capital of type I is sufficient not only for relative over-accumulation but for absolute over-accumulation of capital of type II as well if $\hat{a} \ge 0$.

Relative over-accumulation does not imply absolute over-accumulation of type II. However, the former leads the latter, both last approximately the same duration of time in Z-1 (Table 10).

The relative over-accumulation begins on boom phase and ends at the closing stages of the depression phase. In our simulation run, with one quarter lag absolute over-accumulation of type II starts. The drop of surplus value begins in the final stages of a boom, continues on the phases of the crisis, depression and ends at the beginning of the recovery.

Absolute over-accumulation of type II leads absolute over-accrual of type I; absolute over-accumulation of type I lasts much less than absolute over-accumulation of type II and relative over-accumulation. Absolute over-accumulation of type I covers last quarter of boom and phase of the crisis; the previous local $M_{\max}$ is overcome at the initial stage of the boom, and one quarter before the end its new high is reached (Figure 7, Table 10).

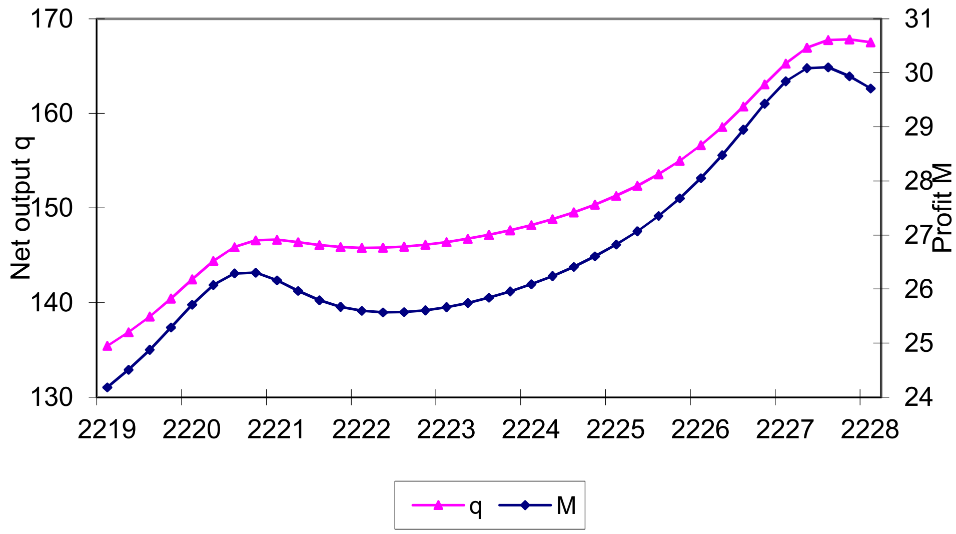

Figure 7 – Highs of profit M lead highs of net product q by one quarter in Z-1, 2219–2228

After reaching $u_{\min}$ simultaneously with $v_{\max}$, $s_{\min}$ and $S_{\max}$, on the boom phase, profit rate is maximal and begins to decline. The absolute decline in profit ends in the first quarter of depression whereas decline of profit rate stops at the end of depression; surplus value sinks to the bottom in the late phase of depression and in the early phase of recovery. Employment rate v reaches the floor during recovery, and the ceiling three quarters prior to the completion of boom along with minimal relative wage u; the maximal relative wage is reached at the end of depression. Notice that $(u_{\min}, u_{\max})$ correspond to $(v_{\max}, v_{\min})$ in Z-1 unlike M-1.

Per capita consumption of workers vw is highest at the end of boom and it is lowest in the early stage of crisis. The previous maximum is achieved in the early stage of recovery, after that growth continues until the end of next boom. Consequently, the improvement of living standard of the working class is, if using the figurative expression of Marx, a stormy petrel for the new crisis.

From cycle to cycle per capita consumption of workers wv grows mostly due to the long-term growth of output per worker a. Wage w is rising throughout the cycle (there is lack of realism in Z-1 in this aspect!).

Growth rate of employment ratio $\hat{v}$ outpaces employment ratio v by one phase, or a quarter-cycle. Maximal $\hat{v}$ is reached one quarter ahead of highs of v, R, $\hat{q}$ and $\hat{k}$, two quarters – ahead of maximal $\hat{a}$.

The more workers produce surplus value, the faster their wages increase. This model relationship corresponds, in our view, to the Marxist theoretical status of use-value of the labour power as the unique source of surplus value. We see that use-value plays outstanding role in political economy! Elevated employment l pushes record profit M down in boom, and resurgent profit M "drags" employment l, suffered from lack of demand, up in recovery. Phases of S and $\hat{w}$ almost matches (Figure 8, Table 10) with some leadership of the first.

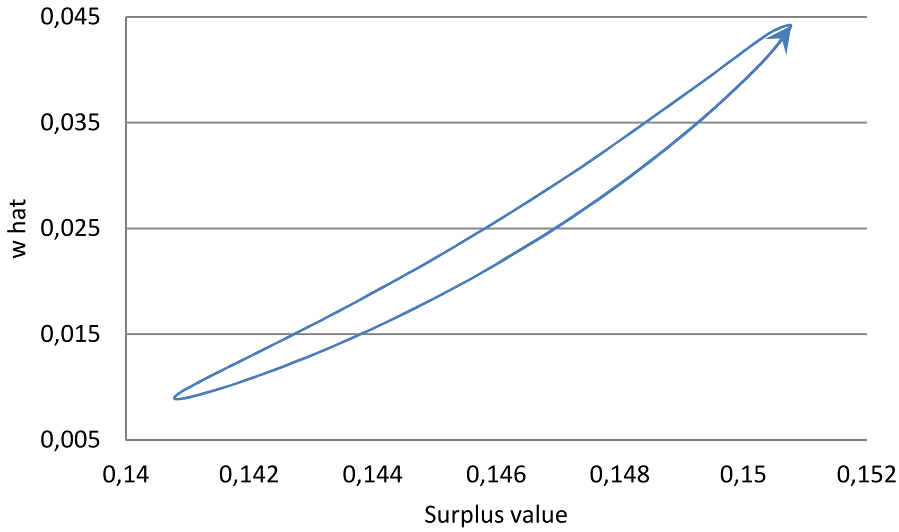

Figure 8 – Surplus value S as a factor of growth rate of wage $\hat{w}$ in Z-1, counter-clockwise, 2219–2227

Relative over-accumulation of capital comes after the 2nd quarter of boom. One quarter later a cyclical maximal surplus value $S_{\max}$ is achieved, employment ratio v also becomes maximal, and then immediately the absolute over-accumulation of capital of type II starts. At a late boom stage profit peaks at a cyclical maximum $M_{\max}$ and immediately absolute over-accumulation of capital of type I manifests itself. Very soon after that (through 1–2 quarters) the economy enters crisis (Table 10). It is on the phases of recovery and boom the three considered forms of over-accumulation of capital are overcome, and capital accumulation finally temporally accelerates.

Figures 11 and 13 illustrate alleged socio-economic harmony in P-2 for the same segment of time, characterized by jerky movements of main cyclical indicators in Z-1 (Figures 9, 10 and 12). Z-1 does not guarantee capitalist reproduction on constant or increasing scale.

For $b = b_0 + 3$ trajectory on the phase plane converge to a limit cycle, which lasts about 6.75 y., for $b < b_0 + 3$ they converge towards stable focus or node; for b belonging to $b_0 + 3 \leq b \leq b_0 + 7.875 = 62.27$ and growing amplitude of fluctuations on transition to closed orbits increases and reaches economically permissible maximum, when $b = b_0 + 7.875 = 62.27$ ($z_{max} = 1$), for $b > 62.27$ fluctuations with greater amplitudes go out the economic region ($z_{max} > 1$).

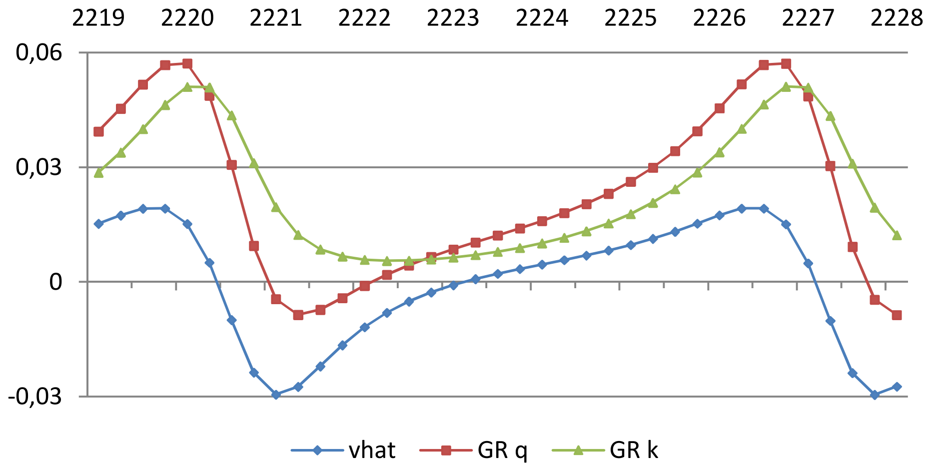

Figure 9 – Growth rates of employment ratio, net product and fixed production assets in Z-1, 2219–2228

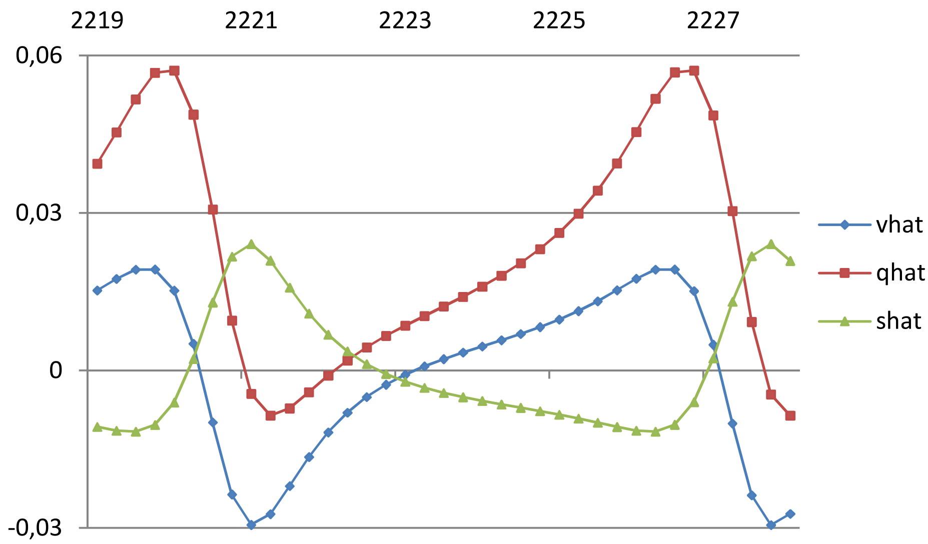

Figure 10 – Over-production, idle fixed capital and unemployment in crisis:
growth rates of employment ratio, net product and capital-output ratio in Z-1, 2219–2228

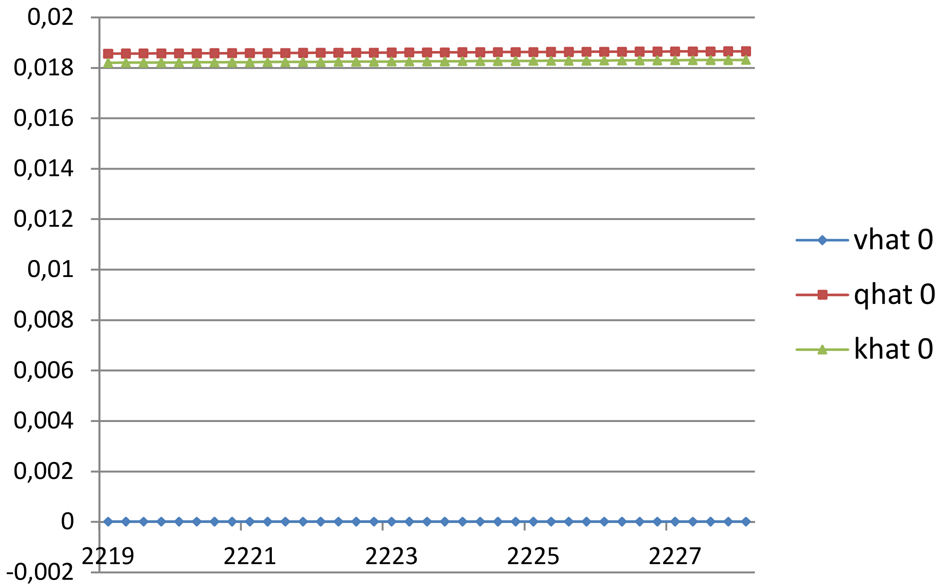

Figure 11 – Growth rates of employment ratio, net product and fixed production assets in P-2, 2219–2228

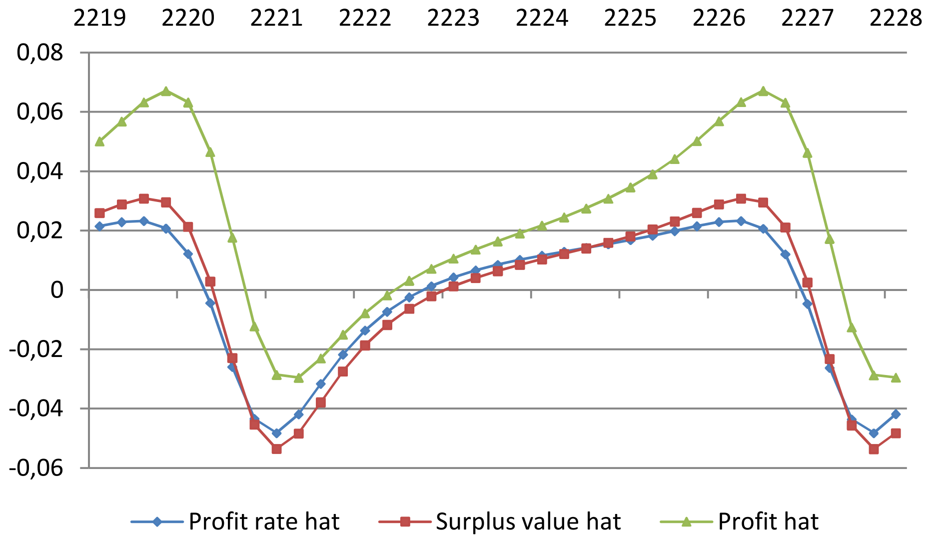

Figure 12 – Occurrence of relative and absolute over-accumulation of capital of types I and II in Z-1: growth rates of profit rate, surplus value and profit, 2219–2228

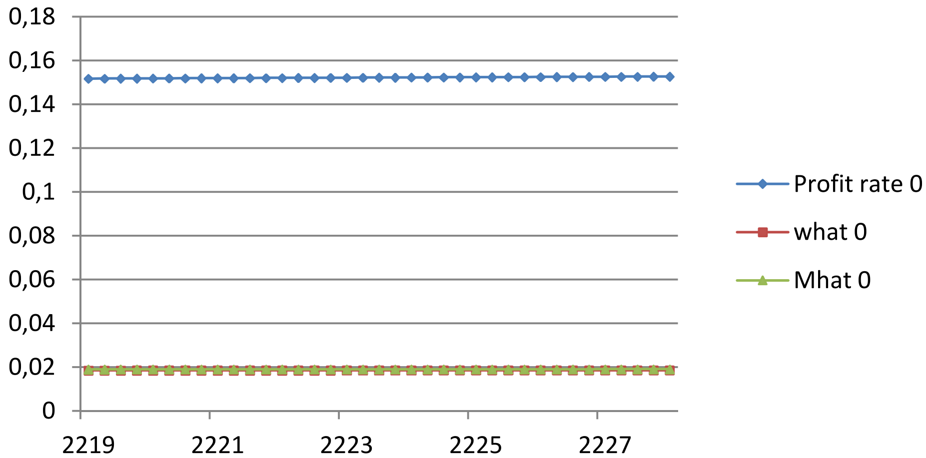

Figure 13 – Socio-economic harmony in P-2: profit rate and growth rates of profit and wage, 2219–2228

We can describe the worst-case scenario where positive feedback loops (Figures 4 and 6) dominate over the negative feedback loops ($b > b_0 + 7.875$). Such domination leads to collapse without prudent stabilization policies not modelled in this paper. In particular, the stabilization policy elaborated in [13, 24] could be effectively applied. This policy could raise a long term employment ratio to a target higher than stationary ones in M-1, P-1, P-2 and Z-1 without lowering a long-term relative wage.

The discredited efficiency wage hypothesis, underlining P-1, P-2 and Z-1, is the particular Achilles heel of these models and should be overcome in the subsequent research. It neglects a forcible reduction of wages as the means attempted for cheapening commodities and for increasing profitability [1].

5. Conclusion

This paper rejects J. Robinson's [16] pessimistic conclusion on the relevance of K. Marx's industrial cycle theory. Similar to [17], the present paper disagrees with A. Atkinson's [14] denial of Goodwin's [7] relevance for business cycle theory. Employing modifications deeper than those suggested by A. Atkinson and later in [17] our modified model Z-1 passes the test earlier suggested by G. Low [15] on the length of trade cycle period. It is demonstrated that at a critical positive magnitude of parameter b a super-critical Andronov – Hopf bifurcation happens and a closed orbit (limit cycle) is generated in the phase space with a sufficiently shorter period than a period of conservative closed orbits in M-1.

As this paper demonstrates further, P-1 and P-2 exclude the existence of big cycles in capitalist economy, terminologically related to the names of S. Kuznets and N.D. Kondratiev in the literature, not to mention their neglecting self-sustained industrial cycles. Short-term fluctuations in inventory stocks and orders are also not considered.

In these models, intensification of competition of workers for jobs strengthens stability of capitalist reproduction while increasing profitability, on the one hand, and reducing relative wage, on the other. Strengthening workers' struggle for higher relative wage, weakening their competition for jobs are destabilizing factors for capitalist reproduction on the increasing scale.

We have also established that [4] contains the unintended significant result: the "neoclassical" rule of net output functional distribution between labour and capital from P-1 [5] is not structurally stable (better to say is conceptually fragile) in P-2 with the additional scale effects. The latter model does not even attempt to justify social distribution of net product between labour and capital by the apologetic "marginal productivity" rule.[7]

Despite some realism uncovered, P-1 and P-2 are empirically and logically controversial. In particular, these "neoclassical" models with a CES production function in terms of k and l_e do not reproduce the positive association of growth rates of net product and output per worker, the prevalent tendencies of relative labour compensation and rate of accumulation to fall, endogenous cycles and other regularities of capitalist economy [11–13, 19, 23, 26].

Extended model Z-1 reflects periodic recurrence of relative and absolute over-accumulation of capital as well as general over-production, as immanent characteristics of industrial cycle. It is shown that capitalists' investment cooperation weakens (competition strengthens) stability of stationary state in Z-1. Targeted reduction of the rate of accumulation increases profit rate and reduces value of labour power contrary to the working class interests.

Contrasting with the textbook Solow (1956)-like "neoclassical" models the employment ratio is not steadily maintained at a constant level in P-1, P-2 and especially in Z-1. Even the stationary rate of unemployment is not "natural" – like the unemployment itself, it is the product of the capitalist mode of production, not of Nature.

The unreliable efficiency wage hypothesis, inherited from P-1 and P-2, remains as structural element within Z-1. Therefore Z-1 is to be liberated from this hypothesis and other remaining conceptual weakness (including a CES production function in terms of k and l_e, wage – "marginal labour productivity" rudiment equivalence and the like) in subsequent research relying essentially on models developed in [11–13, 26].

The flaws in the indigenous "neoclassical" models are mostly due to mixing notions of concrete labour and abstract labour, as well as because of general negation of the K. Marx theory of commodity and surplus value [1]. The "neoclassical" school and some its "radical" supporters stubbornly reject this theory.

The revealed contradictions between the vulgar concepts and reality as well as the structural specification errors identified in the main "neoclassical" equations violate one of the fundamental prerequisites for the intelligent application of regression and other econometric methods [21, 26]. Thus, a necessary preliminary step for subsequent serious statistical investigations has been accomplished.

Non-random, rooted in the material capitalist class interests, nature of the revealed "neoclassical" contradictions and deep-seated specification errors should be further considered in thorough econometric studies. Scientifically advanced university courses on micro- and macroeconomics ought to be liberated from the bourgeois apologetics hidden under superficial and pretentious objectivism.

Marx' methodology is congruent with the methodology of vanguard system dynamics. By learning the predecessor's work seriously and without prejudice, the system dynamics field can flourish further faster and better. This will be a stronger factor of progressive change in the social production relations.

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[7] A similar unintended result has been masked in [25] as [26] demonstrates answering on subjective objections raised in [27]. The models presented in [13, 26] refine and encompass the L. Boggio's [25] "neoclassical" (very contradictory) model with alleged increasing returns.

Appendix A

A proof of Proposition 6.

Jacoby matrix J_a corresponds to stationary state E_a (44). It is defined as

$$J_a = \begin{array}{|c|c|} \hline \frac{(1-\rho)(1+\delta)}{(1-\rho)+\delta}\,\gamma\,\frac{z}{s_a}\,u_a > 0 & f'(v_a)\frac{\delta}{(1-\rho)+\delta}u_a > 0 \\ \hline \frac{z}{s_a}\left[\,(\gamma-1) - \frac{1-\rho}{(1-\rho)+\delta}\,\frac{\gamma}{1-u_{eq}}\right]\frac{1+\delta}{\delta}\,v_a < 0 & -\frac{1}{(1-\rho)+\delta}\,\frac{f'(v_a)}{1-u_a}v_a < 0 \\ \hline \end{array}\,. \tag{59}$$

Notice that Jacoby matrix J_p corresponding to stationary state E_p (31) in P-1 is a special case of (59) for $\gamma = 0$. The reader can use it for checking propositions on LAS of E_p.

A characteristic equation based on (59) is

$$\lambda^2 + b_1\lambda + b_2 = 0, \tag{60}$$

where

$$b_1 = -Trace(J_a) = -\frac{(1-\rho)(1+\delta)}{(1-\rho)+\delta}\,\gamma\,\frac{z}{s_a}\,u_a + \frac{1}{(1-\rho)+\delta}\,\frac{f'(v_a)}{1-u_a}\,v_a\,, \tag{61}$$

$$b_2 = \left|J_a\right| = \frac{z}{s_a}\,u_a v_a\, f'(v_a)\frac{1}{(1-\rho)+\delta}\,(1+\delta)\,(1-\gamma) > 0. \tag{62}$$

Notation $\left|J_a\right|$ is for determinant of Jacoby matrix J_a.

The characteristic equation (60) has the following two roots with a negative real part for typical parameters' magnitudes

$$\lambda_{1,2} = \frac{Trace(J_a)}{2} \pm \sqrt{\left(\frac{Trace(J_a)}{2}\right)^2 - \left|J_a\right|}\,. \tag{63}$$

They are real if initially $\left(\frac{Trace(J_a)}{2}\right)^2 - \left|J_a\right| \geq 0$. Then the stationary state is stable node. Otherwise it is stable focus with a period approximated by

$$T_{P-2} = 2\pi \Big/ \sqrt{\left|J_a\right| - \left(\frac{Trace(J_a)}{2}\right)^2}\;. \tag{64}$$

A proof of Proposition 7. Let $Z_b = Z - z_b$. Jacoby matrix for stationary state E_b in Z-1 is defined as

$$J_b = \begin{array}{|c|c|c|} \hline \frac{(1-\rho)(1+\delta)}{(1-\rho)+\delta}\,\gamma\,\frac{z_b}{s_b}\,u_b & f'(v_b)\frac{\delta u_b}{(1-\rho)+\delta} & -(1-\rho)\,\frac{d}{z_b}\,\frac{\gamma\delta u_b}{(1-\rho)+\delta} \\ \hline \frac{z_b}{s_b}\left(\gamma - 1 - \frac{1-\rho}{(1-\rho)+\delta}\,\frac{\gamma}{1-u_{eq}}\right)\frac{1+\delta}{\delta}\,v_b & -\frac{f'(v_b)}{(1-\rho)+\delta}\,\frac{v_b}{1-u_b} & (1-\gamma)\,\frac{dv_b}{z_b} + \frac{\gamma(1-\rho)}{(1-\rho)+\delta}\,\frac{v_b}{s_b} \\ \hline -b\,\frac{\gamma u_b}{1-u_b}\,\frac{(1-\rho)(1+\delta)}{(1-\rho)+\delta}\,\frac{z_b}{s_b}\,z_b Z_b & -\frac{bf'(v_b)}{1-u_{eq}}\,\frac{\delta u_b z_b Z_b}{(1-\rho)+\delta} & \frac{b(1-\rho)\gamma\delta u_b}{(1-\rho)+\delta}\,\frac{z_b}{s_b}\,Z_b - p \\ \hline \end{array}\,. \tag{65}$$

The standard characteristic equation of the third order is written as

$$\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0, \tag{66}$$

where the parameters are calculated based on the corresponding values of some Jacobi matrix J_X

$$a_0 = -|J_X| = -(J_{11}\ J_{22}\ J_{33} + J_{12}\ J_{23}\ J_{31} + J_{21}\ J_{32}\ J_{13} - J_{13}\ J_{22}\ J_{31} - J_{23}\ J_{32}\ J_{11} - J_{12}\ J_{21}\ J_{33}),$$

$$a_1 = -[J_{23}\ J_{32}\ +\ J_{12}\ J_{21}\ +\ J_{13}\ J_{31} - J_{11}\ (J_{22}\ +\ J_{33}) -\ J_{22}\ J_{33}],$$

$$a_2 = -Trace(J_X) = -(J_{11} + J_{22}\ +\ J_{33}).$$

The parameters of the characteristic equation based on (65) are defined as

$$a_0(b) = a_0 = p\frac{z_b}{s_b}(1-\gamma)f'(v_b)v_b\frac{1+\delta}{(1-\rho)+\delta}u_b > 0, \tag{67}$$

$$a_1(b) = [\frac{1+\delta}{(1-\rho)+\delta}u_b\frac{z_b}{s_b}\ (1-\gamma)f'(v_b)v_b + p\frac{1}{(1-\rho)+\delta}\frac{f'(v_b)}{1-u_b}v_b - p\frac{(1-\rho)(1+\delta)}{(1-\rho)+\delta}\gamma\frac{z_b}{s_b}u_b] +$$

$$+\frac{z_b}{s_b}\ (1-\gamma)f'(v_b)v_b\frac{\delta}{(1-\rho)+\delta}u_b\,(Z - z_b)b = e + ob, \tag{68}$$

where $o > 0$;

$$a_2(b) = -\gamma\frac{z_b}{s_b}\frac{(1-\rho)(1+\delta)}{(1-\rho)+\delta}u_b + \frac{1}{(1-\rho)+\delta}\frac{f'(v_b)}{1-u_b}v_b + p - \gamma\frac{z_b}{s_b}u_b\frac{(1-\rho)}{(1-\rho)+\delta}\delta(Z - z_b)b =$$

$$= c - hb, \tag{69}$$

where $h > 0$.

Lemma 1. The quadratic equation based on the above characteristic polynomial

$$a(b) = a_1(b)a_2(b) - a_0 = 0, \tag{70}$$

where

$$a_1(b) = e + ob, \tag{71}$$

$$a_2(b)\ = c - hb, \tag{72}$$

$$b_1 = -\frac{e}{o} < 0 \tag{73}$$

$$b_2 = \frac{c}{h} > 0, \tag{74}$$

always has two real roots:

$$b_{0,3} = \frac{oc - eh \pm\sqrt{(oc-eh)^2 - 4oh(a_0 - ec)}}{2oh}. \tag{75}$$

Lemma 2. It is true that $-\infty < b_1 < \min(b_3, b_0) \le \max(b_3, b_0) < b_2 < \infty$.

Corollary. The conjugate roots of the quadratic equation $a(b) = 0$ are $b_3 \in (b_1, b_2)$ and $b_3 \le b_0 \in (b_1, b_2)$. It follows from economic requirements that $b_0 \in (0, b_2)$.

The Routh – Hurwitz necessary and sufficient conditions for LAS of E_b in the linearized system are satisfied for $0 \le b < b_0$: $a_0 > 0$, $a_2(b) > 0$ and $a_1(b)a_2(b) > a_0$. As E_b is hyperbolic and LAS, it is LAS also in the non-linear system, q.e.d.

A proof of Proposition 8. Parameter b engaged in equation (51) serves as the bifurcation (control) parameter. Consider the stationary state E_b of the system (39), (40) and (51) as dependent on b:

$$\dot{x} = 0 = f(x, b). \tag{76}$$

The determinant of the Jacoby matrix J_b (65) evaluated at the stationary state E_b (52) differs from zero in our case for any possible stationary state (x_b) as $a_0 = const > 0$ (independently of b). There exists a unique stationary state x_b as changes of b do not affect E_b.

It is assumed the following properties are satisfied:

(a) the components of the function $f(x, b)$, corresponding to the system (39), (40) and (51), are analytic (i.e. given by power series);

(b) the Jacoby matrix $J_b(b_0)$ has a pair of pure imaginary eigenvalues and no other eigenvalues with zero real parts (in this case $\lambda_1 = -a_2(b_0) < 0$);

(c) the derivative $\dfrac{d(\mathrm{Re}\lambda_{2,3}(b))}{db} = \dfrac{-(a_1' a_2 + a_1 a_2') + a_0'}{2(a_1 + a_2^2)} > 0$ (it is the transversality condition);

(d) the stationary state E_b is LAS (for $0 \le b < b_0$).

Then, according to the Hopf theorem, there exists some periodic solution bifurcating from $x_b(b_0)$ at $b = b_0$ and the period of fluctuations is about $2\pi/\beta_0$ ($\beta_0 = \lambda_2(b_0)/i$). If a closed orbit is an attractor, it is called a *limit cycle*. The Hopf theorem establishes only the existence of closed orbits in a neighbourhood of x_b at b_0, still it does not clarify the stability of orbits, which may arise on either side of b_0.

Applying information from the proof of Proposition 7, we establish that conditions (a), (b), (d) of the Hopf theorem are satisfied at $b = b_0$. In particular, the characteristic polynomial for $b = b_0$ is

$$\lambda^3 + a_2(b_0)\lambda^2 + a_1(b_0)\lambda + a_0 = \lambda^2[\lambda + a_2(b_0)] + a_1(b_0)[\lambda + a_2(b_0)]$$

$$= [\lambda + a_2(b_0)][\lambda^2 + a_1(b_0)] = 0. \tag{77}$$

It has the following roots:

$$\lambda_1 = -a_2(b_0) < 0\,; \tag{78}$$

$$\lambda_{2,3} = \pm i\sqrt{a_1(b_0)}\,. \tag{79}$$

It remains only to check that transversality condition (c) is also satisfied. Indeed, for $b = b_0$, $a_1(b) = e + ob$ and $a_2(b) = c - hb$,

$$\frac{d(\mathrm{Re}\lambda_{2,3}(b))}{db} = \frac{-(oa_2 - ha_1) + 0}{2(a_1 + a_2^2)} \approx \frac{ha_1}{2a_1} = \frac{h}{2} > 0 \tag{80}$$

as $oa_2(b_0) \approx 0$, q.e.d. A magnitude of this derivative equals 0.018 in our simulation run.

The supercritical character of the Andronov – Hopf bifurcation has been established only experimentally in multiple simulation runs. An analytical proof of this property still remains a Hercules challenge. The tools from [28] could be appropriate "ammunition".

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