Dollars and Sense II: Our Interest in Interest, Managing Savings, and Debt

Lesson 1

Can Compound Interest Work for Me?

Instructions for Teachers

Overview of Contents

This lesson contains three "hands-on" and progressively challenging simulations designed to let students EXPLORE and EXPERIENCE the system of compounding interest, what Albert Einstein is alleged to have called "the most powerful force in the universe."

- Simulation 1 offers students a "simple" 7-Day Challenge: predict the final value of an allowance fund that, starting with $1 on Day 1, is matched daily (between $1 and $5 for each $1 in the fund) over the next 6 days;

- Simulation 2 lets students explore more "realistic" scenarios involving an interest rate (between 1 and 5%) on a Savings Account opened with a one-time deposit for a number of years (from 1 to 20); and, finally,

- Simulation 3 encourages students to compare options for building a $25,000 college fund using different starting ages (when the student is between 1 and 17), interest rates (1-10%), and rates of annual deposits.

In each case, students will see the results of different compounding interest scenarios plotted out over time in GRAPHS and TABLES.

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MATERIALS

- Computer Simulation (available online at http://www.clexchange.org/curriculum/dollarsandsense/Dollars and Sense II/ds2_lesson1.asp.

- Three handouts (use as needed) to record plans and results.

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Core Objectives For Lesson 1

(1) <u>Compound Interest</u>. The core financial lesson take-home message is this: the "power" of compound interest earned on one's savings rests with the amount of money deposited in a savings account, the interest rate paid on those savings, and the amount of time the account accrues interest.

(2) <u>Using Models to Test Options.</u> The open-ended and hands-on focus of each of the three simulations in this lesson are designed to encourage students to explore and evaluate different options and opportunities in identifying and subsequently explaining to others a personal preference. At the core of this process is a recognition that there is no single right answer for everyone. Rather, there are options, trade-offs, and ultimately multiple pathways through which students can define and subsequently achieve personal financial goals.

(3) <u>How Compound Interest Works.</u> In addition to supporting a mathematical understanding (using equations, in addition to graphs and tables) of the compounding process, students are presented with systems thinking conceptual tools to bolster their understanding of interest as a "System" whose core structure explains its behavior (exponential growth).

Deepening Understanding For How The "System" Works

The conceptual tools of systems thinking help visualize the dynamic process that unfolds over time. The core process of compound interest is represented

as a conceptual system (shown below): the amount of <u>annual interest earned</u> is governed by the amount of money in <u>MY SAVINGS</u> account and a rate of interest (e.g., 4% = .04) paid annually (not shown here) on that SAVINGS.
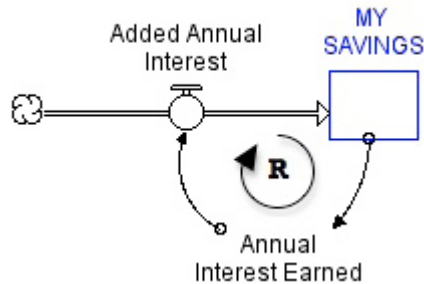
What follows are brief introductions to each of the three simulations, "annotated" versions of suggested student handouts to accompany each of the simulations, and possible follow-up questions and activities for extended learning opportunities.

SIMULATION 1: A 7-Day Mathematical Challenge

http://www.clexchange.org/curriculum/dollarsandsense/
Dollars and Sense II/ds2_lesson1.asp

This exercise is designed for students who are unfamiliar or uncomfortable with solving a compound interest problem using an algebraic compounding equation. Presenting a simple compounding challenge—*Starting with a $1 allowance on day 1, how many $ will you have on day 7 given a daily match of $2 for every $1 you have?*—students are encouraged to solve the problem initially with pencil and paper.

In representing the changing daily match of $$ as a FLOW adding to one's total accumulation—or STOCK—of MY ALLOWANCE $, systems thinking tools help students see how compounding interest—or, in systems terms,

Systems thinking tools help students see how compounding interest—or, in systems terms, reinforcing feedback—works.

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reinforcing feedback—works: the daily "match" adds to the total of MY ALLOWANCE $ which, on the next day, increases the amount of the match, adding more ALLOWANCE $, and an even larger match, and so on.

The simulation itself broadens students' ability to explore "what ifs," using a "Daily Match" between $1 and $5 for each of MY ALLOWANCE $. Seeing multiple results helps students understand the NON-LINEAR dynamics of compound interest, where interest grows faster later than earlier and where differences in the amounts of interest translate into far larger differences over time.

SIMULATION 1 HANDOUT with ANSWERS and GUIDES FOR TEACHERS

Can Compounding Interest Work for Me?
A 7-Day Mathematical Challenge

1. Open the Simulation, read the Introduction, and summarize your task below:

It is important that students understand (and can explain) the learning objective for using the simulation. The task is to compare two allowance payment options, a lump sum ($100) or, starting with $1 on the first day, a $2 match daily for each $ already in the account for a week.

Which option should you choose?

2. Can you solve this using the mathematical table below? (answers in red)

Time	MY ALLOWANCE FUND	New Allowance $ Added Next Day
1	1	2
2	3	6
3	9	18
4	27	54
5	81	162
6	243	486
Final	729	

3. Now use the simulation to explore this and other scenarios. What if your father had offered a different amount of "compounding interest," for instance:
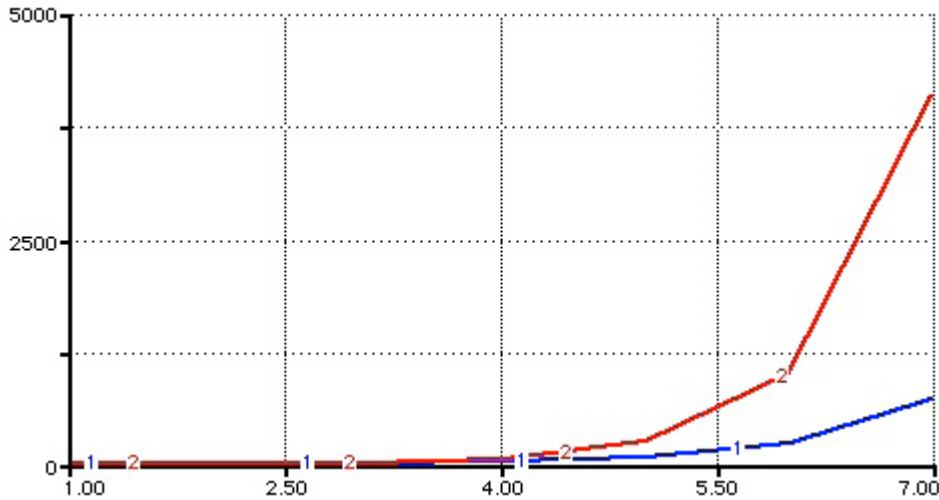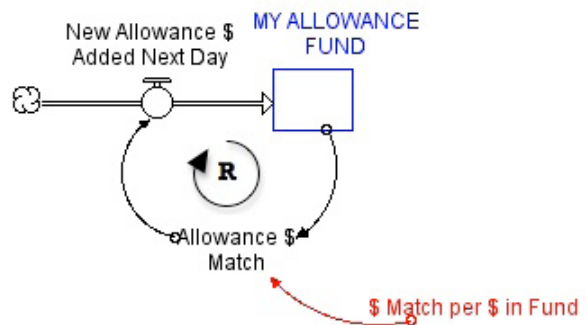
$1: $64 $3: $4,096 $5: $46,556

The key concept to communicate here is the NON-LINEAR nature of compound interest. A doubling (say, from $1 to $2) generates more than 7x the final total Allowance Fund $; a tripling (from $1 to $3) produces more than 60x the total. A graph offers another perspective (illustrated below, with $2 and $3 compounding). That's the power of the compounding process!

MY ALLOWANCE FUND : 1 – 2 –

4. Can you describe mathematically how "allowance $ match" (on right) is calculated each day? Can you create a mathematical equation to solve this or similar problems? (See next page.)

Here, the diagram seeks to focus student attention on the two factors that affect the growing STOCK (or accumulation) of MY ALLOWANCE FUND: (1) the "interest rate" (developed in the previous question) as well as (2) the current number of dollars in MY ALLOWANCE FUND on which interest is calculated. Each day, the STOCK of MY ALLOWANCE FUND grows, which translates into an ever-increasing flow of New Allowance $ added.

For those interested, this may also be a good time to introduce the mathematics of exponential growth. The generic mathematical equation for simple compound interest is defined as:

$$P = C (1+r)^t$$

Where P = future value
C = initial deposit
r = interest rate
t = number of years invested

In this exercise, that equation translates into the following:

Total Allowance $ =
Initial Deposit ($1) x (1 + "Daily Match") to the power of number of days "invested" (or 7 here)

In the case of a daily match of $2, that means:

$= 1 * (1+2)^7$
$= 3^7$
$= 729$

5. Consider a different problem with compounding "interest": Someone starts a rumor about you on Day 1 and tells two people; each day, for a total of 7 days, each of these rumormongers tells two people. How many people will have heard the interesting rumor after 7 days?
TOTAL RUMORMONGERS: ___

Using either a table or the equation developed above:
Total Rumormongers = Initial (1) * (1 + 2 new people)^7 = 729

6. Can you invent a 7-day compounding problem of your own?

Ultimately, the best learning occurs when students apply what they've learned to other "problems" or situations. Ideally, engaging students in this conversation will facilitate that level of learning.

SIMULATION 2: Interest on a Single Deposit

http://www.clexchange.org/curriculum/dollarsandsense/
Dollars and Sense II/ds2_lesson1.asp

This exercise offers a simple introduction to the real world system of compounding interest on savings. Starting with a word problem ("If I put $100 in a bank account or bought a Certificate of Deposit (CD) earning 4% annual

interest, how much will I have in 8 years? Or 12 or 15? What if I start with $225? $382? $1,250?"), the lesson offers both a mathematical equation and a hands-on simulation, the latter providing opportunities for exploring additional scenarios of interest. GRAPHICAL output of MY SAVINGS is supplemented with TABULAR output showing how annual interest adds to SAVINGS, thus leading to more interest, still higher SAVINGS and so on. This simple exercise shows how the "power" of compound interest reflects the amount of principle (or SAVINGS), the annual interest rate, and—above all else—the importance of TIME.

SIMULATION 2 HANDOUT with ANSWERS and GUIDES FOR TEACHERS

Can Compounding Interest Work for Me?
Calculating Interest Based on a Single Deposit

1. Open the Simulation, read the Introduction, and summarize your task below:

It is important that students understand (and can explain) the learning objective for using the simulation: The task is to compare interest earned on a single deposit in a Savings account.

2. Can you solve these problems using the mathematical equation below?

$100 at 4% interest/8yrs: $136.86

$100 at 4% interest/12yrs: $160.10

$100 at 4% interest/15yrs: $180.09

$225 at 4% interest/8yrs: $307.93

$225 at 6% interest/8yrs: $358.62

Simple Compound Interest Equation
(where interest is compounded only once per year)
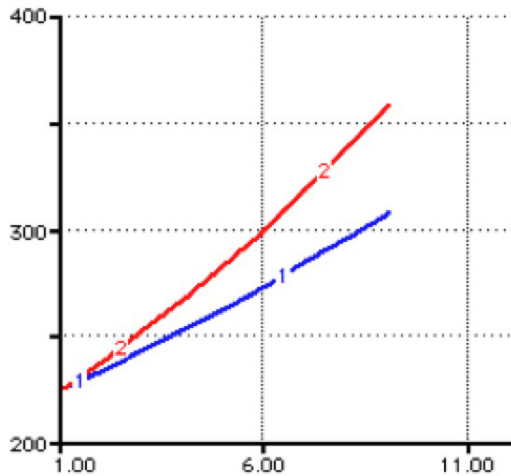
$P = C (1+r)^t$
Where P = future value
C = initial deposit
r = interest rate (expressed as decimal: e.g., 0.06)
t = number of years invested

3. Use the simulation to check your results.

Note that the graph provides an opportunity to compare different strategies, as shown below (with a $225 deposit earning 4% versus 6% for 8 years), while the table more clearly shows how annual interest adds to Savings. Ideally, both help inform the role of interest rates, together with TIME (and Deposit amount!) in creating the "power" of the compounding process.

Time	MY SAVINGS P	Added Annual Interest
1	$225.00	13.50
2	$238.50	14.31
3	$252.81	15.17
4	$267.98	16.08
5	$284.06	17.04
6	$301.10	18.07
7	$319.17	19.15
8	$338.32	20.30
Final	$358.62	

4. Now use the simulation to explore a personal scenario of interest ("What if?"). Identify a GOAL and a PLAN for generating Savings using a single deposit.

Amount of Single Deposit: $_____   GOAL: $_____

Explore plan options for achieving the GOAL. Select the most appealing option and describe how and why it will work.

This is an open-ended exercise, following up on what students have learned about the three core factors that underlie the "power" of compound interest. Students should have a clear GOAL in mind, and should be encouraged to explore (and share) different strategies for achieving it. Describing their "choice" provides a vehicle for them to explain both what they've learned and to highlight different options and trade-offs.

Interest Rate: _____ %   Time: _____

Why it is my choice: _____

_____

SIMULATION 3: Options for Building College Savings

http://www.clexchange.org/curriculum/dollarsandsense/
Dollars and Sense II/ds2_lesson1.asp

The engaging question here is simple: Is it possible to avoid borrowing for college? Using a target of $25,000 —the average debt for current graduates—this exercise invites students to "educate" parents on the importance of saving early and regularly, and again recognizes the "power" of compounding interest over years. The simulation offers students many strategies for achieving their financial goal. Rather than offering a "right answer," it strives to engage students (and parents) in evaluating options and choosing one that best works for them. The simulation can also be used to explore how similar decisions influence retirement funds—more interesting to parents and teachers, perhaps, than students!

> Students record their PLANS without focusing on one "right" answer.

SIMULATION 3 HANDOUT with ANSWERS and GUIDES FOR TEACHERS

Can Compounding Interest Work for Me? Building College Savings

1. Open the Simulation, read the Introduction, and summarize your task below:

It is important that students understand (and can explain) the learning objective for using the simulation. The task is to see how a parent(s) can avoid borrowing for college by saving $25,000 before their child enters college.

2. Compare two plans, one of which involves saving at birth (or year 1), another at age 10. Prepare plans that include the following:

	Plan A	Plan B
a. When (what age) to start saving?	1	10
b. How much to put in SAVINGS to start?	___	___
c. How many years to save?	___	___
d. How much to deposit each year?	___	___
e. At what age does the child start college?	___	___
f. What is the likely annual interest rate?	___	___
g. How often each year is interest compounded?	___	___

The focus here is primarily on TIME. That is, the earlier one starts saving for college, the less of one's own money it will take to reach the $25,000 goal.
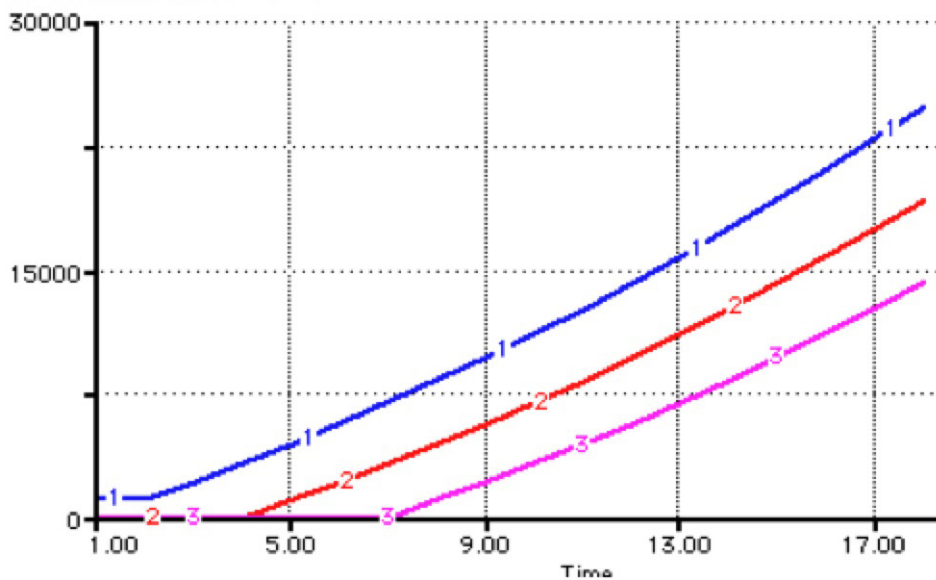
3. Next, use the simulation to explore these and other options.

For illustrative purposes, using a 4% interest rate, annual deposits of $1,050 starting at birth will reach that goal; with a similar interest rate of 4%, and savings starting at age 10, deposits are closer to $2,750 per year.

Perhaps a more interesting exploration—one of many—involves starting the $1,050 payments at ages 1, 4, and 7 and comparing the results on the graph (see below). Note that total interest differs by more than $4,600. That's significant!

There are any number of "right answers" for achieving the $25,000 savings goal, and students should be encouraged to explore and compare multiple strategies.

COLLEGE SAVINGS FUND : 1 – 2 – 3 –

4. Print the option that you propose to share with a parent and be prepared to justify your choice.

Ultimately, the challenge here is advocating for one strategy over any number of others. The quality of that advocacy is based both on the comprehensiveness of their exploration (Have they considered all of the options that "fit" with their savings expectations?) and their recognition of compound interest as a desirable part of their plan.

5. Finally, identify that factor or factors (a – g listed on page 9) that are most important for achieving your PLAN. Explain their importance below.

_____

_____

Here, students should be able to compare the relative impact of deposits, interest rate, and TIME in shaping their PLAN. Where deposits start early, interest should be a more substantial contributor, with that reinforced by higher interest rates. Again, this provides an opportunity for students to evaluate their plan.

6. What if you were saving for retirement? Can you apply what you've learned with the College Fund in identifying the best strategy for building a healthy Retirement Fund, say at age 65?

The core structure of a Retirement Fund is that of a Savings Fund. As such, this College Savings model can easily be converted into a Retirement Fund by altering the presumed starting date (as early as 18? or later?) and the "Age Starting Retirement" (65? 70?). Ask students: Given what you've learned from College Savings, what would be a wise strategy (again, to advise your parents, perhaps?) for maximizing a Retirement Fund? Then have them explore…

Summary Challenge (after completing the lesson)

Students are encouraged, after completing each of the simulations, to apply what they've learned to address a meaningful, real world savings problem of personal interest. (Suggested written options are included with the handouts.) This challenge obliges them to ground their understanding of how the system works with realistic decisions regarding deposits, interest rates, time, and costs. Sharing their plan engages others in constructive discussion of options and choices.
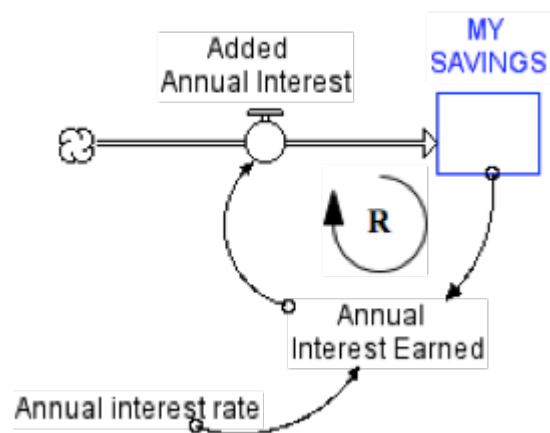
SUMMARY CHALLENGE HANDOUT with GUIDES FOR TEACHERS
Summary Challenge (after completing the lesson)

Pick #1 or #2 and write your answer in the space below (add graph or table, if desired):

1) Identify something that you intend to save for to buy in either 5, 10, or 20 years. Can you use what you've learned in this lesson to describe how you'll do so, and why your plan will be successful? (Make sure it's realistic!) Explain.

2) Substitute a friend or family member in Question #1. Identify something they hope to buy in 5-20 years and use what you've learned to help them create a realistic plan with which they will be successful. Explain.

In this final exercise, students are challenged to apply what they've learned to address a meaningful, real world savings problem of personal interest. In asking them to think long-term, they need to think about how, when, and what they'll make for deposits, together with researching realistic interest rates that strive to maximize interest payments. They'll also need to consider the likely cost of something at the time they're ready to buy it (e.g., college costs are always rising!).

The goal here is not to generate THE right answer but to challenge the student to explain A right answer that best addresses their needs and capabilities. Challenging them to explain their plan to others provides opportunities to engage others in similarly applying what they've learned.

Can Compounding Interest Work for Me?
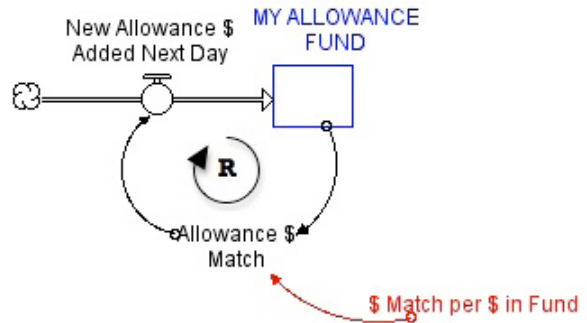## A 7-Day Mathematical Challenge

1. **Open the Simulation, read the Introduction, and summarize your task below:**

2. **Can you solve this using the mathematical table below?**

Time	MY ALLOWANCE FUND	New Allowance $ Added Next Day
1	1	
2		
3		
4		
5		
6		
Final		

3. **Now use the simulation to explore this and other scenarios. What if your father had offered a different amount of "compounding interest," for instance:**

    $1: _____ $3: _____ $5: _____

4. **Can you describe mathematically how "Allowance $ Match" (on right) is calculated each day? Can you create a mathematical equation to solve this or similar problems? (See one example below.)**

    _____

    _____

5. **Consider a different problem with compounding "interest": Someone starts a rumor about you on Day 1 and tells two people. Each day for a total of 7 days, each of these rumormongers tells two people.**
    How many people will have heard the interesting rumor after 7 days?

    TOTAL RUMORMONGERS: _____

6. **Can you invent a 7-day compounding problem of your own?**

Can Compounding Interest Work for Me?
Calculating Interest Based on a Single Deposit

1. **Open the Simulation, read the Introduction, and summarize your task below:**

2. **Can you solve these problems using the mathematical equation below?**

Simple Compound Interest Equation
(where interest is compounded only once per year)

$$P = C (1+r)^t$$

Where P = future value
C = initial deposit
r = interest rate (expressed as decimal: e.g., 0.06)
t = number of years invested

$100 at 4% interest/8yrs: _____

$100 at 4% interest/12yrs: _____

$100 at 4% interest/15yrs: _____

$225 at 4% interest/8yrs: _____

$225 at 6% interest/8yrs: _____

3. **Use the simulation to check your results.**

4. **Now use the simulation to explore a personal scenario of interest ("What if"). Identify a GOAL and a PLAN for generating Savings using a single deposit.**

    Amount of Single Deposit: $_____ GOAL: $_____

    Explore plan options for achieving the GOAL. Select the most appealing option and describe how and why it will work.

    Interest Rate: _____ %   Time: _____

    Why it is my Choice:

Can Compounding Interest Work for Me?
Building College Savings

1. **Open the Simulation, read the Introduction, and summarize your task below:**

2. **Compare two plans, one of which involves saving at birth (or year 1), another at age ten. Prepare plans that include the following:**

		Plan A	Plan B
a.	-- When (what age) to start saving?	1	10
b.	-- How much to put in SAVINGS to start?	___	___
c.	-- How many years to save?	___	___
d.	-- How much to deposit each year?	___	___
e.	-- At what age does the child start college?	___	___
f.	-- What is the likely annual interest rate?	___	___
g.	–- How often each year is interest compounded?	___	___

3. **Next, use the simulation to explore these and other options.**

4. **Print the option that you propose to share with a parent and be prepared to justify your choice.**

5. **Finally, identify that factor or factors (a – g listed above) that are most important for achieving your PLAN. Explain their importance below.**

6. **What if you were saving for retirement? Can you apply what you've learned with the college fund to identify the best strategy for building a healthy retirement fund at age 65?**

Summary Challenge (after completing the lesson)

Pick #1 or #2 and write your answer in the space below (add graph or table, if desired):

1. Identify something that you intend to save for to buy in either 5, 10, or 20 years. Can you use what you've learned in this lesson to describe how you'll do so, and why your plan will be successful? (Make sure it's realistic!) Explain.

2. Substitute a friend or family member in Question #1. Identify something they hope to buy in 5-20 years and use what you've learned to help them create a realistic plan with which they will be successful. Explain.

Lesson 1   Can Compound Interest Work for Me? • 19
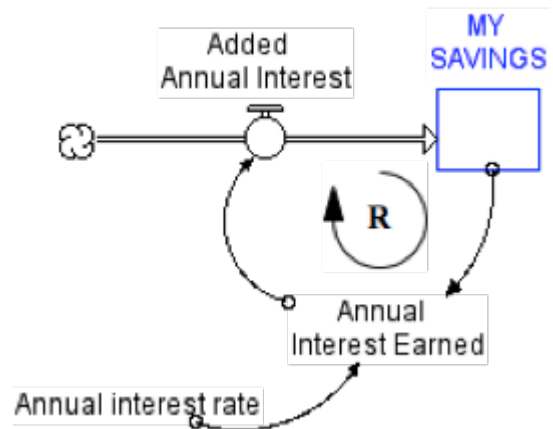
Lesson Title(s):
Dollars and Sense II, Lesson 1: Can Compound Interest Work for Me?
Dollars and Sense II, Lesson 2: How Can I Maximize Savings While Spending?

Overview:
The simulations in *Dollars and Sense II* introduce 6th – 12th grade students to the terminology and basic structures of *compound interest* and how it relates to saving and spending. Later simulations in this series also include *interest payments* on debt. Students become aware of the influence of time in the calculation of interest, both as it helps (in the case of savings) and hurts (in the case of debt).

Related Characteristic(s) of Complex Systems:
Conflicts arise between short-term and long-term goals.

Ideas and Examples for Connecting to the Characteristic:
Lesson 1 of the *Dollars and Sense II* series revisits the concept of exponential growth (also covered in D&S) through examples and exercises using compounding interest.

In Lesson 2, five simulations build understanding of the increasing role of interest as a source of savings when the savings timeframe is long. In the short term, the most benefit comes from managing spending because interest on savings is negligible. Over the long term, interest on savings becomes more important. Of course, money spent is not saved, so both *savings* and *time* are needed to maximize the benefits of compound interest.

The underlying heart of these simulations is the concept of the time value of money. It can be a difficult idea to grasp. Some ideas to develop understanding are:

1. Ask students to interview parents and grandparents about the cost of large and small purchases when they were young. Chart the responses as a class. They may be shocked to learn that a pack of gum cost five cents or a house could be purchased for $20,000. Over time, prices generally increase (inflation), so the same amount of money buys less in the future. Earning interest is a way to "keep up" with inflation.
2. Although prices generally increase over time to buy new items, most purchases have a useful life and thus their value decreases. Have students compare the prices of new cars with the same models that are five years old. Values can easily be found on Kelley Blue Book: http://www.kbb.com/.

Resource(s)
A video that covers time-value concepts in student-friendly terms:
http://www.youtube.com/watch?v=Dux1D-QZoLU

An interesting take on decision-making as applied to purchases:
http://www.youtube.com/watch?v=65mNGYereX8